AN INVESTIGATION OF THREE SAMPLED-DATA BANDPASS FILTERS

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FILTERS

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PREFACE

Imagine, for a moment, the complexities that might be avoided in radio receiver design if a return could be made to the tuned-radio-frequency (TRF) designs so popular in the early days of radio. Such a vintage - 1930 receiver would, of course, not be adequate for use in today's crowded radio bands. The design itself, however, is an excellent one in that it avoids the spurious signal difficulties associated with the modern multiple conversion superhetrodyne. If some technique could be found that would enable sufficient selectivity to be introduced into the basic TRF receiver, then a return to this design would indeed seem desirable. One possible approach to achieving this selectivity will be discussed in this thesis.

It is difficult for me to adequately express my appreciation for the aid rendered in carrying on this investigation. Texas Instruments, Inc., Dallas, Texas, acted as sponsors for the project and was most helpful in supplying much technical advice and most of the materials required in the work. Mr. Walter Matzen, of Texas Instruments, was of special help. I am deeply indebted to my major adviser, Dr. Harold T. Fristoe, for his help and encouragement throughout the project, and to Freddie

Wenninger for his many suggestions pertaining to transistor circuit design. I would like also to express my appreciation to Dr. William L. Hughes, Head, School of Electrical Engineering, Oklahoma State University, without whose interest this work would not have been possible.

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CHAPTER I

INTRODUCTION

Who, today, is not familiar with the action of a radio receiver as it is tuned across the standard broadcast band. As the tuning dial is turned from one extremity to the other, one station becomes audible, fades out, and is replaced by the next station on the dial. Such is a prime example of the action of a bandpass filter. It has the property of being able to separate a desired signal, or group of signals, from among a number of undesired transmissions.

Ideally, a bandpass filter has properties as shown in Figure 1-1. Any signals falling in the frequency range from $f_o - f_c$ to $f_o + f_c$ are passed unattenuated; signals falling outside this range are attenuated completely. The frequency range for which signals are passed unattenuated is commonly called the passband of the filter. Notice that the ideal (analytic) bandpass filter is characterized by a perfectly rectangular passband. Ideal bandpass filters for certain specialized applications may have characteristics differing from this.

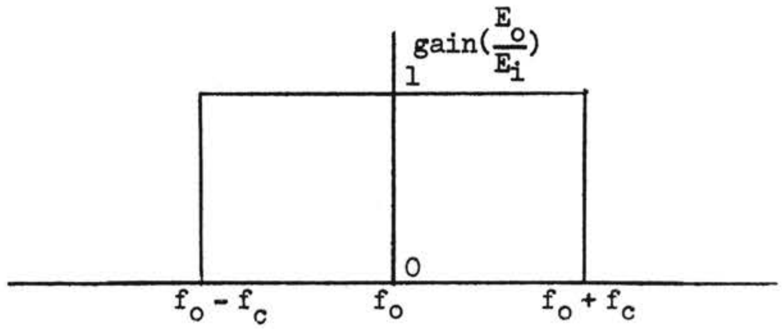

Figure 1-1. Characteristics of an Ideal Bandpass Filter

Practical filters may be said only to approach the ideal. They lack the infinitely steep sides and the perfectly flat top of the ideal filter and often will introduce some attenuation into the passband. Many different types of bandpass filters have been constructed, but since they all have equivalent electrical circuits, it is instructive to first mention the simple L-C parallel resonant circuit, Figure 1-2.

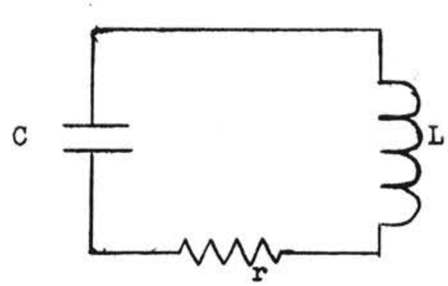

Figure 1-2. Simple Parallel Resonant Circuit

The bandpass characteristics of such a circuit may be represented by the following equation from Langford-Smith (1):

$$\frac{A_o}{A} = \sqrt{1 + Q^2\left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2} \qquad (1\text{-}1)$$

where f_o is the center, or resonant, frequency of the filter, f is the frequency in question and Q is the quality factor of the circuit given by:

$$Q = \frac{\omega_o L}{r} = \frac{2\pi f_o L}{r} \qquad (1\text{-}2)$$

Equation 1-1 leads to a bandpass characteristic as shown in Figure 1-3. It may be seen that it bears little resemblance to the ideal characteristic of Figure 1-1. Figure 1-3 also shows the effect the quality factor has on the shape of the passband.

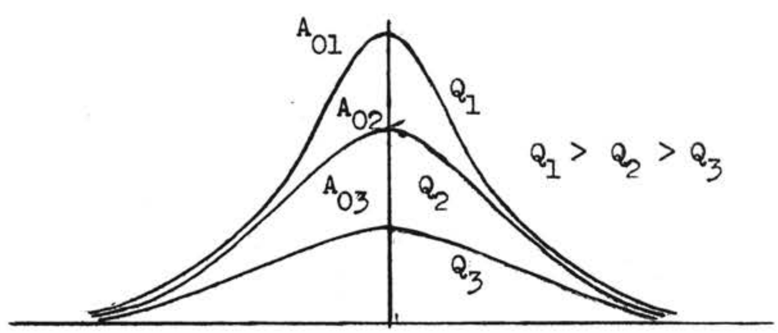

Figure 1-3. Bandpass Characteristic of a Simple L-C Parallel Resonant Circuit

Increasing Q leads to a decreasing bandwidth at any given center frequency, or conversly, it may be said that increasing Q leads to increased selectivity (that is, the ability of the tuned circuit to select one signal from among many). Selectivity is defined as the bandwidth of the filter at some specified attenuation. It is common to make this measurement at the so-called "half-power" points on the curve, or where the voltage gain (**A**) falls to $\left(\frac{1}{\sqrt{2}}\right)$ of the center frequency gain (A_o) but it is not uncommon to hear discussions of "skirt selectivity" which refer to a similar measurement made 50 or 60 decibels down the curve.

It is possible to define another term which yields a measure of the "squareness" of the passband. Shape factor may be defined as the ratio of the selectivity 60 decibels down to the selectivity 6 decibels down.

$$S.F. = \frac{BW_{60\ db}}{BW_{6\ db}} \quad . \tag{1-3}$$

The ideal filter of Figure 1-1 yields a shape factor of one. Practical circuits yield a factor higher than one. The higher the factor, the less the filter under consideration approximates the ideal.

On a strictly qualitative basis, it may be seen that the shape factor for the simple L-C circuit is relatively poor due to its broad skirts. Much can be done to generate filters with narrower skirts, however. Techniques such as overcoupling two tuned circuits with the same f_o or using a number of tuned circuits with slightly different center frequencies (stagger tuning) are two common solutions which achieve decreased shape factors at the expense of simplicity and ease of adjustment (1).

In recent years, new techniques have made possible low shape factors even in relatively simple circuitry. Quartz crystals have long been used to achieve high selectivity in radio receivers (1). In their most common application, however, they yield a non-symmetrical passband which, although highly useful in certain applications, does not approach the desired ideal passband. It is possible to use these crystals in circuits which do give nearly ideal shape factors (2). Such filters have been packaged and sold as "black-box" units.*

Other materials have also been investigated for similar use. Ferrites have been investigated by Roberts (3) and others and a number of companies have made available a device with excellent shape factor using a series of very high Q metallic discs.** At least one manufacturer has made use of a ceramic bandpass determining element***

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*Hycon Eastern Inc., 74 Cambridge Parkway, Cambridge 42, Mass., Burnell and Co., Pelham, N.Y., and others.

**Collins Radio, 19700 San Joaquin Road, Newport Beach, Calif., and others.

***Clevite Electronic Components; Division of Clevite Corporation, Cleveland 14, Ohio.

operating much in the manner of the crystal filters mentioned above in a transistorized communications receiver (4). The ceramic element was chosen for this application because of its compatibility with low impedance transistor circuitry.

All of these devices are similar in that they may be analyzed and discussed in terms of electrical analogs using only passive elements. They all reduce to combinations of simple L-C tuned circuits and are, therefore, dependent upon circuit Q in achieving whatever characteristics they may have.

Now, Q may also be defined in terms of the center frequency and bandwidth of the filter,

$$Q = \frac{f_o}{BW} \tag{1-4}$$

where BW is the bandwidth 3 decibels below the response at the center frequency. It is obvious from this expression that the Q required to give a specified selectivity (in cycles-per-second) goes up directly with the center frequency of the filter. Increasing the center frequency of the filter by a factor of ten also increases the Q required to give the specified selectivity by the same factor. This is the reason that it has been necessary to incorporate crystal and mechanical elements in high quality bandpass filters operating at relatively high frequencies (above 400 kilocycles). Only these elements can provide the Q necessary to achieve the filter characteristics required for applications in modern communications equipment.

This same simple relation has led receiver manufacturers to use double and, in some cases, even triple conversion to meet performance specifications in modern communications receivers. In a superheterodyne receiver it is desirable to use as high an intermediate frequency as possible in order to minimize image response while achieving high selectivity demands exactly the opposite. The compromise has been to use two intermediate frequencies with suitable frequency conversion between them to meet both specifications (5). This leads to considerable design complexity and the introduction of a number of difficult to control spurious responses from the extra frequency conversion used. Other complexities occur because of the necessity of having selectivity ahead of the first radio-frequency stage of the receiver also (1). Since this is not a paper on receiver design difficulties, it is not possible to go further into this subject here. It should be obvious, however, that the ramifications of Equations 1-3 and 1-4 have led to much complexity in receiver circuit design.

It should also be pointed out that as bandpass filters are made to approach the ideal their complexity also increases and it becomes a practical impossibility to tune them. That is, to vary their center frequency without in some way also affecting the passband characteristics.

These complexities have led circuit designers to seek out new avenues on which to approach the problem of bandpass filter design. One very interesting approach is the technique to be discussed in this thesis. With the introduction of appropriate non-linear processes into the filter design, it becomes possible to bring the low-pass bandpass analogy often used in circuit design into practice. The end result is a filter with independent center frequency and bandwidth. Since the passband characteristics of the filter are determined solely by low-pass filter characteristics, the design of a tunable bandpass filter may be broken down into two phases. The first phase consists of the design (or purchase) of the required number* of low-pass filters with appropriate cut-off frequency and shape factor. Phase Two consists of using these low-pass filters in circuitry which will achieve the desired low-pass bandpass transformation. Since low-pass filter design is usually considerably simpler than the design of an equivalent bandpass filter, this two phase process may greatly simplify the bandpass filter design process while allowing a more nearly ideal filter to be constructed. These filters are usually referred to as "Sampled-Data" bandpass filters, and, in essence, act to translate a low-pass filter characteristic up to and about some center frequency (f_o) much in the same manner that an audio

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*Two or four usually required.

signal is translated up in frequency under amplitude modulation. A comparison is made between the two processes in Figure 1-4.

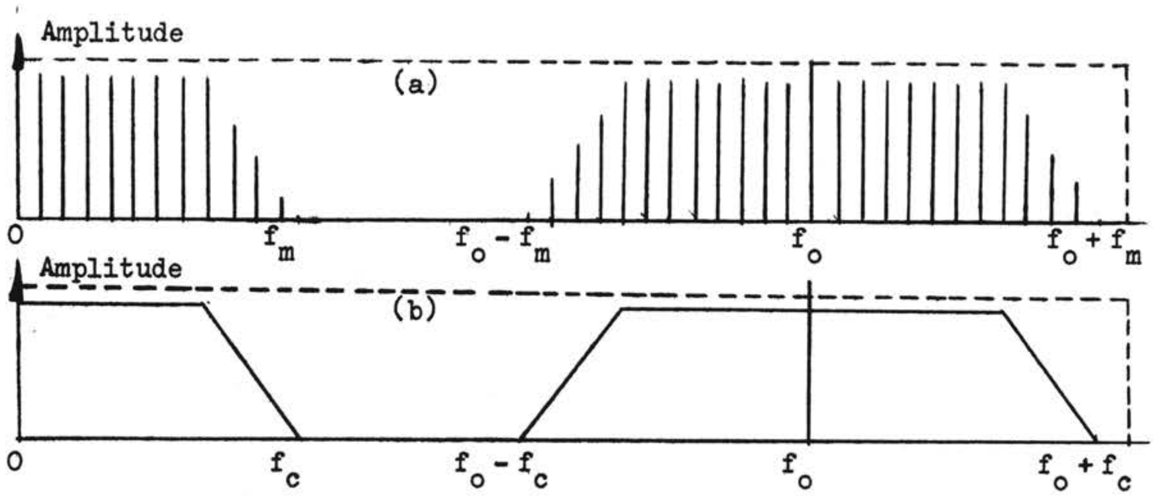

Figure 1-4. (a) Translation of an Audio Spectrum Under Amplitude Modulation
(b) Translation of a Low-Pass Characteristic in a Sampled-Data Filter

In both cases, spurious signal components are generated by the switching processes involved. These spurious components must be removed by additional filtering which is represented by the dotted low-pass characteristics shown in the same figure. The similarity between the two techniques is obvious.

The original work in this field was done just after World War II in England. A polyphase modulation technique was proposed by Barber (6) in which the passband was produced through the use of two local oscillator signals in quadrature feeding appropriate modulators and a pair of identical low-pass filters. Madella (7) in a separate paper points out a possible ambiguous signal situation using this device, but proposes no solution. A further investigation of devices of this type was carried on by MacDiarmid and Tucker (8). This paper also contains information on the then new problem of single sideband detection and generation. In a later paper, Tucker (9) presents a description of a highly selective transmission test set using a passband generation device. A circuit diagram for the test set is shown.

Work in the United States has been primarily concerned with passband generation devices using digital techniques although at least one worker has proposed a device operating on the quadrature modulation principle (10). The operation of these Digital Filters (as the digital device has become known) has been thoroughly investigated by a number of people. Franks and Sandburg (11) have presented a thorough theoretical analysis of the device as well as experimental data on a filter they constructed. Papers concerned with analysis only have been presented by Smith (12), Fischl (13) and Brown, Cahn and LePage (14). The Digital Filter, in its general form of N identical paths, provides a multiplicity of identical passbands. Such a device is called a "comb" filter and has found application in radio direction finding and Doppler radar systems (13). Within the past two years, a number of workers have investigated filter designs of the digital type (11, 15, 16).

The work in this field at Oklahoma State University (Stillwater, Oklahoma) was undertaken at the request of Texas Instruments, Inc. (Dallas, Texas) who was seeking information as to the applicability of a Digital Filter to the tuning element of a standard broadcast band (550 to 1600 kilocycles) receiver. In theory, such a receiver could be extremely simple in design (Figure 1-5) and could be made highly compact by the use of microminiature circuitry. The digital filter is easily applied to this kind of circuitry because of the switching nature of its operation.

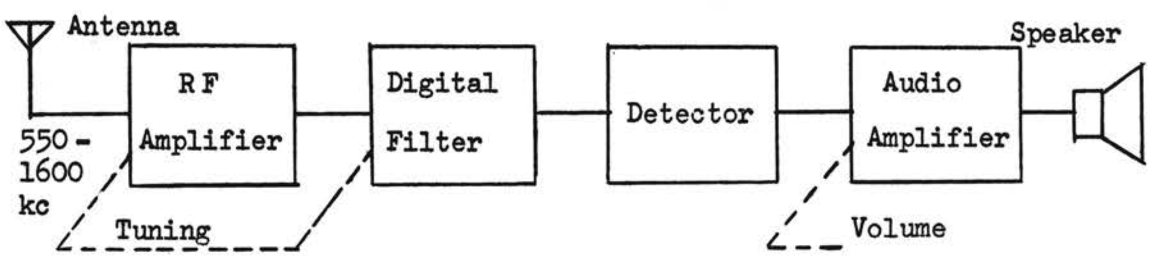

Figure 1-5. Broadcast Receiver Using Digital Filter

As is outlined in Chapter II, the actual construction and operation of such a receiver proved to be more difficult than first imagined because of the high rates and near perfection required of the switching system. Also included in this chapter is a general discussion of the operation of these devices as well as the results of some experiments conducted on a Digital Filter operating at 5 kilocycles.

While working on this receiver project, an apparent similarity between the switched waveforms in the digital filter and typical PAM* waveforms became obvious. This led the author into an investigation of passband generation devices using modulation techniques. A filter, using essentially the same system described by MacDiarmid and Tucker (8) was constructed. Since this device uses only two low-pass paths, no multiplicity of passbands is generated. Only one passband centered about the local oscillator frequency of the filter is produced. An analysis of this device is presented in Chapter III. A complete description of its circuitry (it is fully transistorized) and the results obtained with it are presented in Chapter IV.

In some respects, the operation of this circuit is very similar to the techniques used in both the "phasing" and the so-called "third" method of single sideband generation. Neither of these methods is concerned with the generation of a filter characteristic, however (17, 18).

Working with this filter using quadrature modulating

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*Pulse Amplitude Modulation.

functions led naturally into the Two-Path Digital Filter analyzed in Chapter V. This two path device may be considered as a combination of the two filters discussed previously and has some advantages over either of them in most applications. The circuitry and operation of this filter are discussed in Chapter VI. A comparison between all three Sampled-Data filters is made in Chapter VII.

CHAPTER II

THE DIGITAL FILTER

This chapter is concerned with the digital filter work carried on at Oklahoma State University during the academic year 1962-63. Three different filters are discussed; the first two using single RC low-pass sections but operating at different center frequencies, 5 kilocycles and 1.5 megacycles. The third filter makes use of two RC low-pass sections and a direct-coupled amplifier in each filter path so that the over-all digital filter could be made to exhibit gain. The advantage of such a design was that the gain in the passband (which is at 1.5 megacycles) was obtained without the use of high frequency transistors.

Passband data is included for the 5 kilocycle filter together with a discussion of some of the operating characteristics of the 1.5 megacycle filter. Only enough theory is included that the reader may understand the results of the chapter. He is referred to the references (11, 12, 13, 14, 15, 16) for a more thorough discussion of Digital Filter operation.

In general, a Digital Filter consists of N identical low-pass paths along with some mechanism for switching through the paths sequentially. Such a device is shown in Figure 2-1.

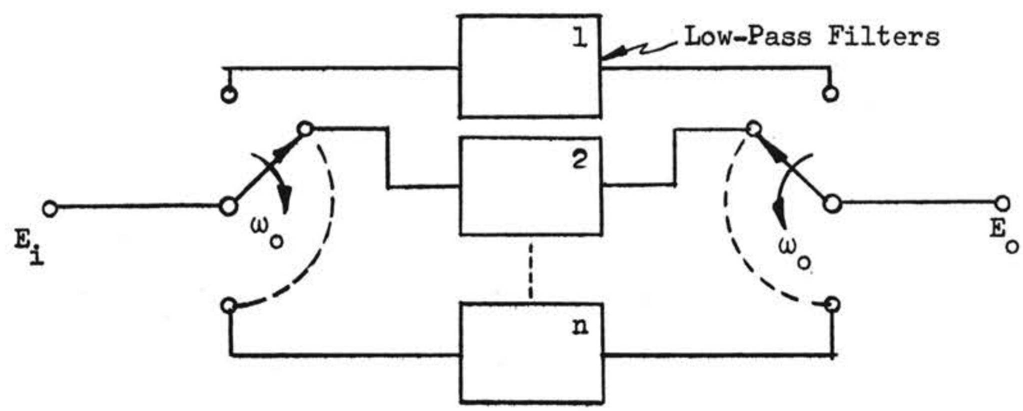


Figure 2-1. General N-Path Digital Filter

The input and output switches are rotated in synchronism at some angular frequency ω_o which becomes the center frequency of the filter. Certain constraints are placed on the characteristics of the device. It is necessary to limit the absolute cutoff frequency (ω_c) of the low-pass filters to less than one-half ω_o to prevent overlapping of the bandpass and low-pass characteristics as shown in Figure 2-2. Since spurious responses are generated at harmonics of ω_o by the switching, it is necessary to place a low-pass filter at the output to eliminate these signals. Similarly, the filter will respond to signals at harmonics of ω_o placed on the input so a filter must be placed at the input to avoid this possibility. The amplitude of the passband generated at ω_o has been computed to be 0.81 of the low frequency gain of the low-pass filter (15).

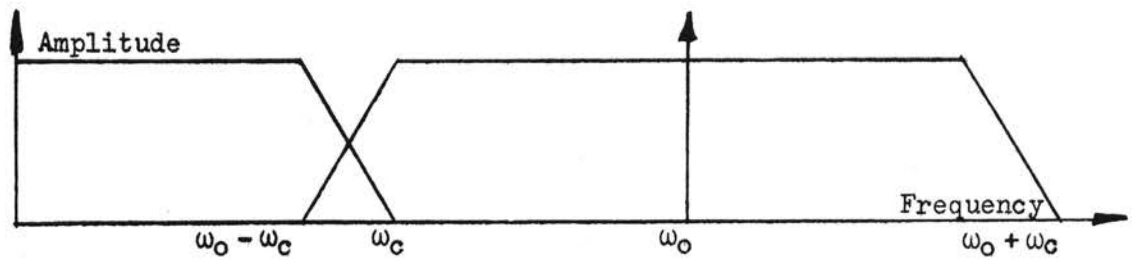


Figure 2-2. Demonstration of Filter Passband Overlap

The result of this synchronous switching is the translational effect mentioned in Chapter I. A passband is generated about the switching frequency (ω_o) having the characteristics of the low-pass filters used in each of the n paths.

It is possible to switch in other ways than as shown in Figure 2-1. One method is to switch a four path filter using four transistors in the clamp configuration shown in Figure 2-3. This circuit is the one used by the author for his 5 kilocycle center frequency filter. Each base is driven by a one-fourth duty cycle pulse derived by combining V_a and V_b in the four possible combinations. The equivalent low-pass configuration for this filter has been previously determined and is shown in Figure 2-4 (15).

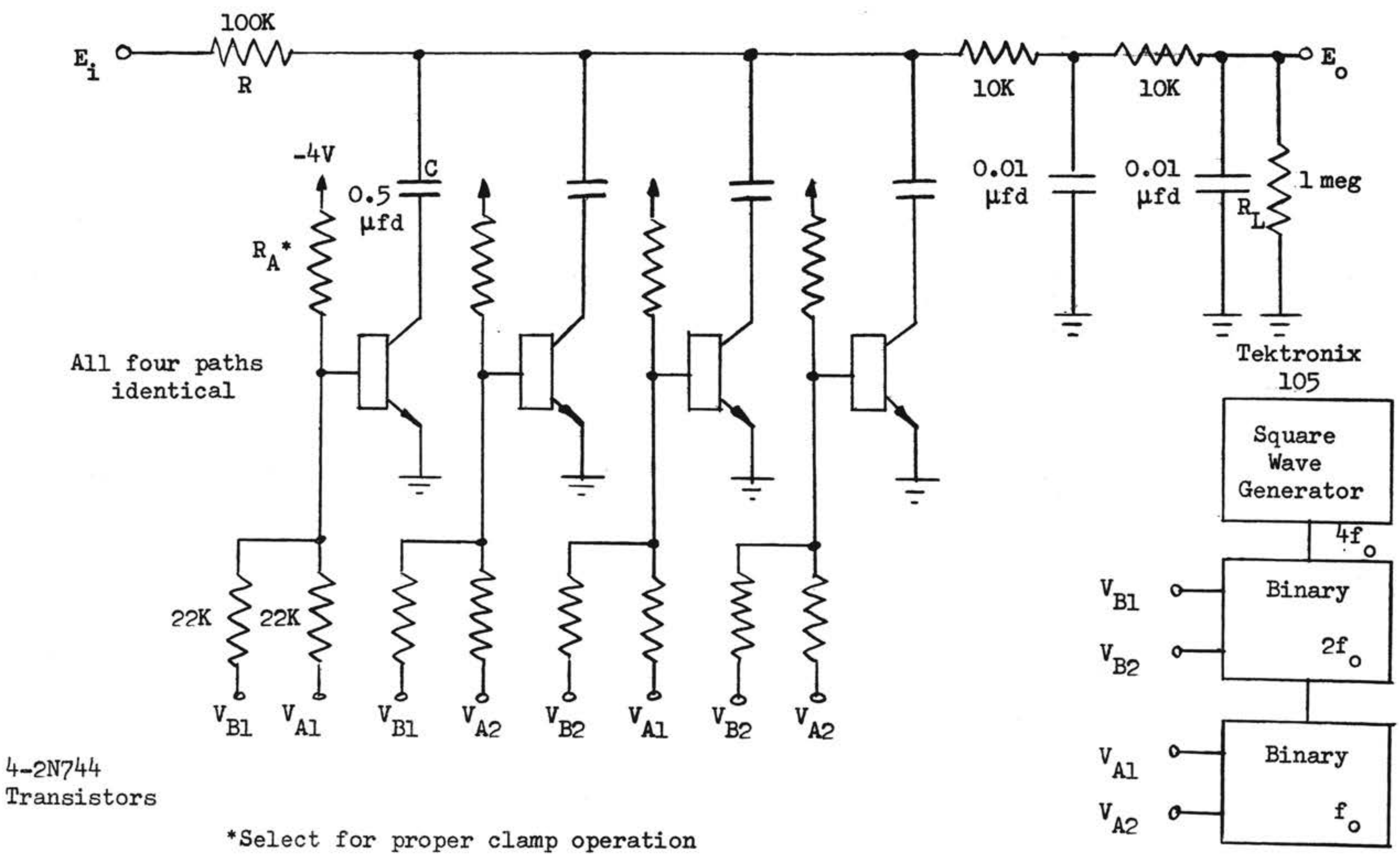


Figure 2-3. Four-Path Digital Filter Switched by Transistors Acting in the Clamp Mode

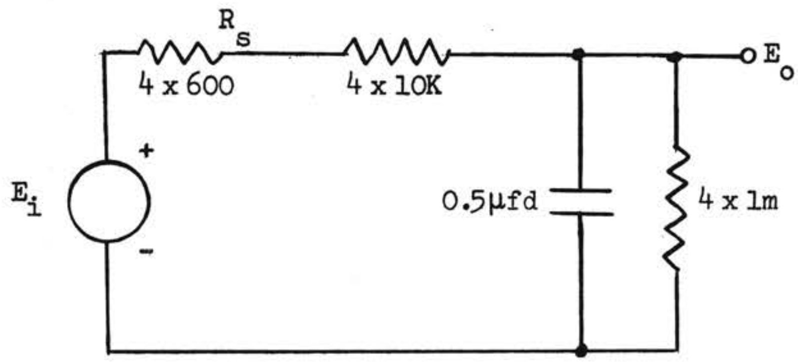


Figure 2-4. Equivalent Low-Pass Network for Digital Filter of Figure 2-3

A simple calculation of the upper breakpoint frequency of this circuit reveals it to be

$$f_c = \frac{1}{2\pi RC} = \frac{1}{6.28(40 + 2.4) \text{ x } 10^3 \text{ x } \frac{1}{2} \text{ x } 10^{-6}} =$$

$$= 7.6 \text{ cps.}$$

The resultant bandwidth of the filter is twice this, or 15.2 cps.

Bandpass data was taken for this filter and is presented in Figure 2-5. In Figure 2-6, it has been normalized and compared to the asymptote computed above. It may be seen that both the cut-off frequency and the rate of roll-off (20db/decade) are as predicted.

Two major difficulties were encountered with this filter. First, it was very difficult to get a low-pass filter with sharp enough roll-off for use on the output so

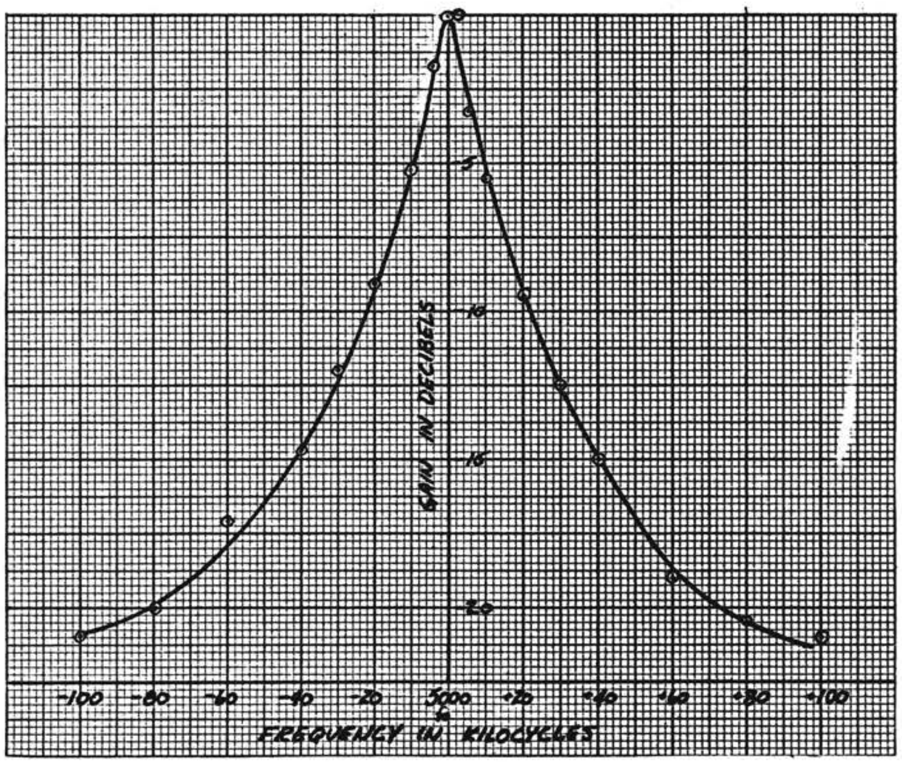


Figure 2-5. Measured Passband for Four-Path Digital Filter of Figure 2-3

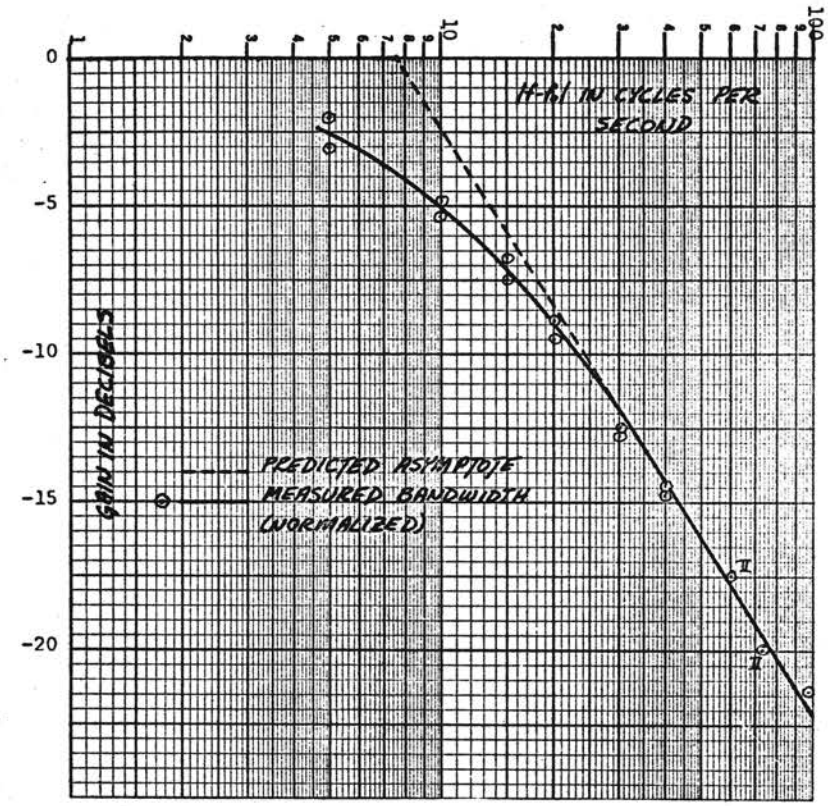


Figure 2-6. Normalized Passband of Filter of Figure 2-3

that the harmonics of ω_0 generated would be suppressed. The author finally compromised on the two section RC filter shown, but this leaves much to be desired. Due to this lack of proper filtering, the output signal was not a pure sine wave but rather a heavily integrated square wave which had a considerable amount of switching noise superimposed on it.

This switching noise was the second major problem. Spikes on the order of 200 millivolts were encountered even with no signal applied and the input and output terminated in 510 and 22K ohms, respectively. The primary cause for these spikes was the switching delays of the binaries used and represents the major difficulty encountered in constructing filters of this type.

When it was decided to construct a filter for the broadcast band, a change was made in the logic circuitry. Figure 2-7 shows the new system used. The astable multiviabrator was made tuneable over the range 1500 to 3000 kilocycles making the center frequency of the filter variable over the 750 to 1500 kilocycle range corresponding roughly to the upper one-half of the standard broadcast band.

As with the previous logic, two square waves with one one-half the frequency of the other were combined to form the required one-fourth duty cycle switching pulse. The gates serve the dual function of squaring the output of the free-running multivibrator and introducing some delay to compensate for the switching delay in the binary. The high speed of this logic system required care in layout and the use of very fast switching transistors (2N744's, 400mc alpha cut-off frequency). Switching times in the 10 to 20 nanosecond range were achieved here. Even so, spikes were obtained in the output of the filter with amplitudes in the vicinity of 200 millivolts peak-to-peak. Again, the binary delay was the primary cause of these transients.

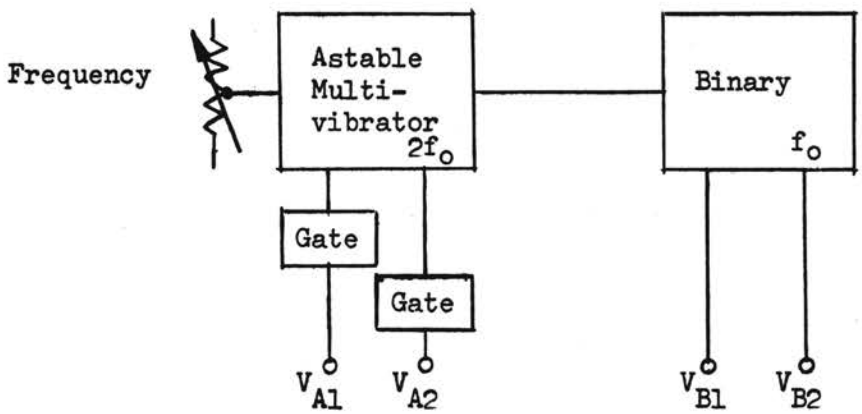


Figure 2-7. Logic for Broadcast Band Digital Filter

Two different types of filters were tried using this logic. The first was exactly the same as the previous model, but with an equivalent low-pass bandwidth of three kilocycles.

It was originally felt that to insure fast turnoff times with the clamp transistors at this frequency special care would have to be taken to keep the series resistance (R_1) and source resistance (R_s) very low (less than 1K). This was found not to be the case, however (see Figure 2-8).

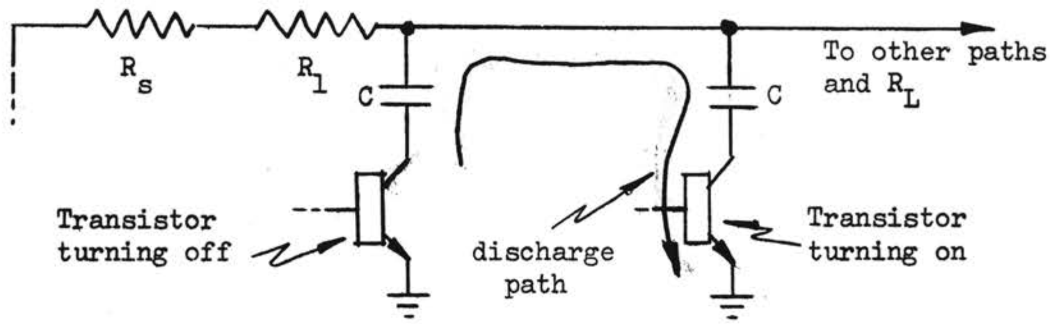


Figure 2-8. Demonstration of Low Impedance Path to Ground in Four-Path Digital Filter

Because the time constant R_1C is very much longer than the switching period, the voltage across each capacitor may be considered constant over this period. Each capacitor C may then be replaced by an ideal voltage source in an equivalent circuit. If now, only one capacitor was being switched (assume the other three are removed from the circuit for the moment), then it would be necessary to keep the parallel combination of $(R_s + R_1)$ and R_L less than 1K. Since the other capacitors are present, and since one of these is always shorted to ground through its corresponding clamp transistor, there will always be a low impedance path for the collector circuit capacities to discharge through. Therefore, it would seem that switching considerations do not enter into the selection of the low-pass filter circuit impedances.

Unfortunately, it was not possible to make accurate bandpass measurements at these frequencies with the equipment on hand. An approximate check, however, showed that the 3 decibel bandwidth was as predicted by the equivalent low-pass network.

Attention was next turned to a completely different filter configuration. It was desired to obtain gain through the Digital Filter by placing amplifiers in each of the low-pass filter paths. The basic configuration is shown in Figure 2-9. Each box marked "LPF" incorporates the circuitry shown in Figure 2-10. Since it was necessary that the low-pass filters have no lower cut-off frequency a two stage direct-coupled amplifier (two differential stages) fed back to a gain of 100 was used. This was followed by a two section RC low-pass filter with a cut-off frequency of about three kilocycles. Switching was accomplished by the two 2N744's acting as clamps. In this situation, the clamp transistors must be held on for three-fourths of the total switching period and released for the remaining one-fourth of the time. The logic system was modified slightly to accomplish this. To prevent the output clamp transistor from disturbing the charge on the 0.05 microfarad filter capacitor, a 1N914 diode was used. This diode opens up whenever the output clamp is

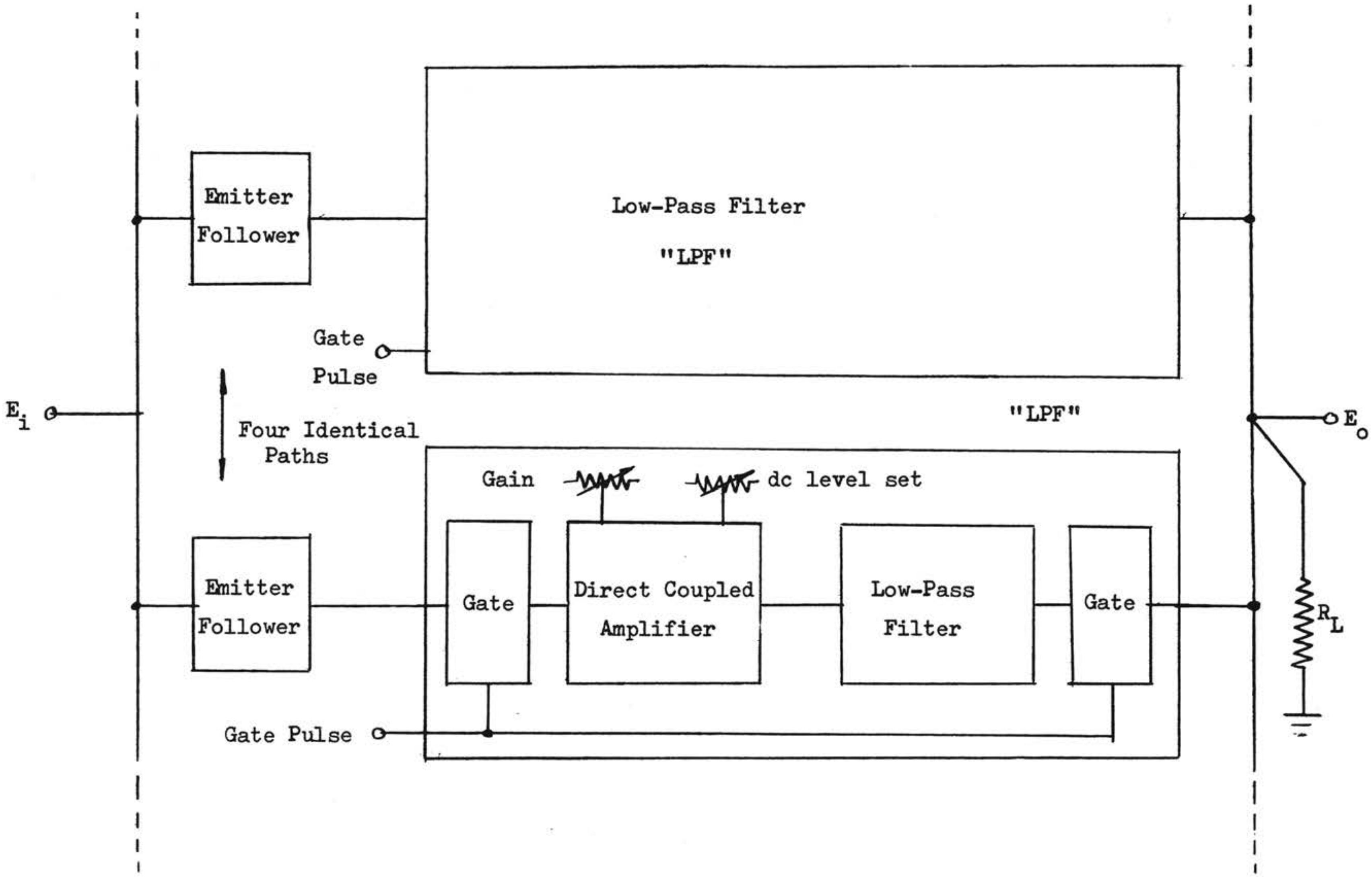
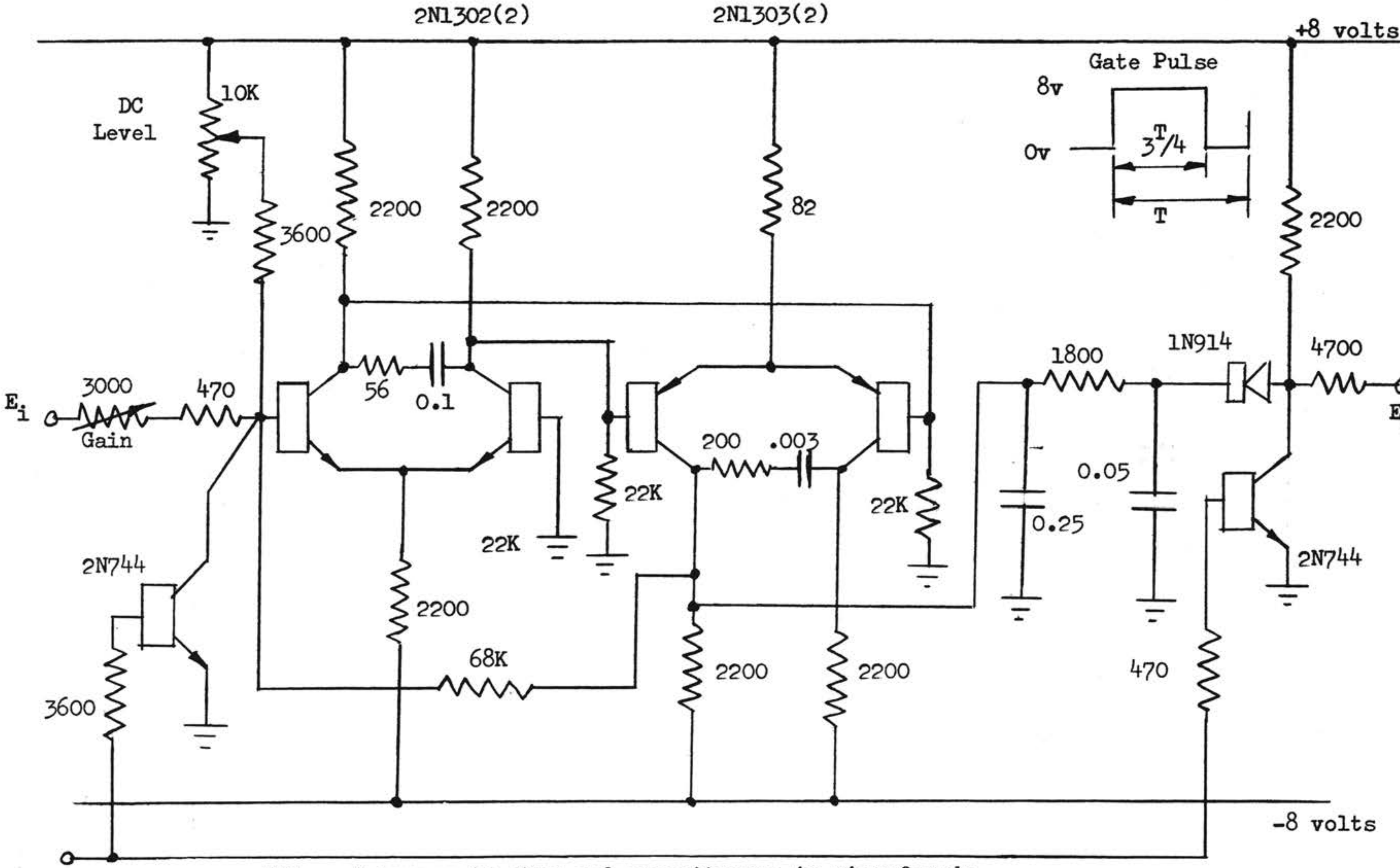


Figure 2-9. Four-Path Digital Filter With Direct Current Amplifiers in Each Path



All resistances in ohms and capacitances in microfarads

Figure 2-10. Schematic of Direct Current Amplifiers and Low-Pass Filters Used in Figure 2-9 held on. The over-all maximum gain of each of these paths was approximately 15.

The bandpass action of the filter was as expected from the theory. Gain was exhibited in the bandpass although it was lower than expected (about three). Once again, it was impossible to make accurate bandpass measurements, but the expected bandwidth was approximately verified.

Both of the filters constructed for the broadcast band operated sufficiently well to verify the predicted operation. In addition, the author has shown that it is possible to construct a Digital Filter that will exhibit gain by including amplifiers in the low-pass paths. A number of problems have become apparent, however. As simple as a Digital Filter receiver may appear on paper (Chapter I, Figure 1-5), the actual mechanization may become quite complex and costly. The high switching speeds required of a Broadcast Band filter may lead the designer into relatively complex circuitry which could require unreasonably large amounts of power for its operation. The author's breadboard model of the last filter mentioned above drew over 250 milliamperes from two 8-volt power supplies. The bulk of this went to the logic circuitry. Using microminiature circuitry and "starved" current techniques would reduce this, of course, but it might not then be possible to obtain the required switching times.

Switching transients were the largest single problem in these filters. At the lower frequencies (in the audio range), it would indeed seem possible to reduce these transients greatly by the use of more sophisticated switching circuits than used here. In the broadcast band, however, the rise and fall times of the switching pulses are an appreciable percentage of the pulse width and unless each of the four gating pulses are properly timed, large transients result. This is true even if special, fast switching transistors (such as the 2N744) are used and switching times in the vicinity of 10 nanoseconds are achieved.

These transients act to limit the small signal handling capabilities of the filter. In order to handle signals on the order of 10 microvolts (which modern receivers are capable of handling), the transients would have to be at least this small (1). Achieving this represents quite a hurdle in the path of constructing a digital receiver.

It would also seem impractical to build a filter with gain by including dc amplifiers in the low-pass filters. In order to obtain an over-all gain of three, it was necessary to construct a direct coupled amplifier (with all the attendant temperature stabilization problems) with a gain in the vicinity of 100. The switching networks absorbed most of this gain.

Readying this filter for operation was also something of a problem. It was necessary to match (using the appropriate controls), both gain and dc output level. This was somewhat time consuming because of a slight interaction between the channels. All in all, it should be said that this does not appear to be a reasonable solution to the problem of obtaining gain in the passband. A straight forward amplifier in front of the filter would appear to be the best solution.

Concurrently with this work, a similar investigation was being carried on by Texas Instruments, Inc., of Dallas, Texas, using microminiature logic and a two section RC low-pass network. Much the same problems as mentioned here were encountered in the Texas Instrument research. The main difference between the two projects was that Texas Instruments used a modulated signal for testing and detected the radio-frequency signal at the output of the Digital Filter with a peak detector. This violates one of the constraints previously mentioned (the low-pass filter on the output) but it avoids the problem of having to construct a low-pass filter with variable breakpoint which will operate effectively over the entire broadcast band. One problem which has appeared is a beat between the input signal and the logic pulses which is visible in the detected output of the filter. This appears to be an inherent problem in the system, but one which may be overcome by careful design.

CHAPTER III

PASSBAND GENERATION USING MODULATION TECHNIQUES - I
(ANALYSIS)

This is the first of two chapters to be devoted to a bandpass filter using quadrature modulation functions. The Bibliography contains a number of relevant references.

The material here provides a basis for the experimental work described in Chapter IV. A physical description of the system is given along with the mathematical basis for its operation.

This system may be most easily explained with reference to the block diagram shown in Figure 3-1.

Two identical low-pass paths are used. In each path, the low-pass filters are preceded by a mixer and followed by a balanced modulator. A local oscillator operating at the center frequency of the filter (ω_0) feeds a signal to each of the mixers and modulators. Ninety degrees phase shift is applied to the local oscillator signals which feed the lower path in the figure. To simplify discussion, the upper path will be referred to as the I, or <u>I</u>n-Phase Channel, and the lower path as the Q, or <u>Q</u>uadrature Phase Channel.

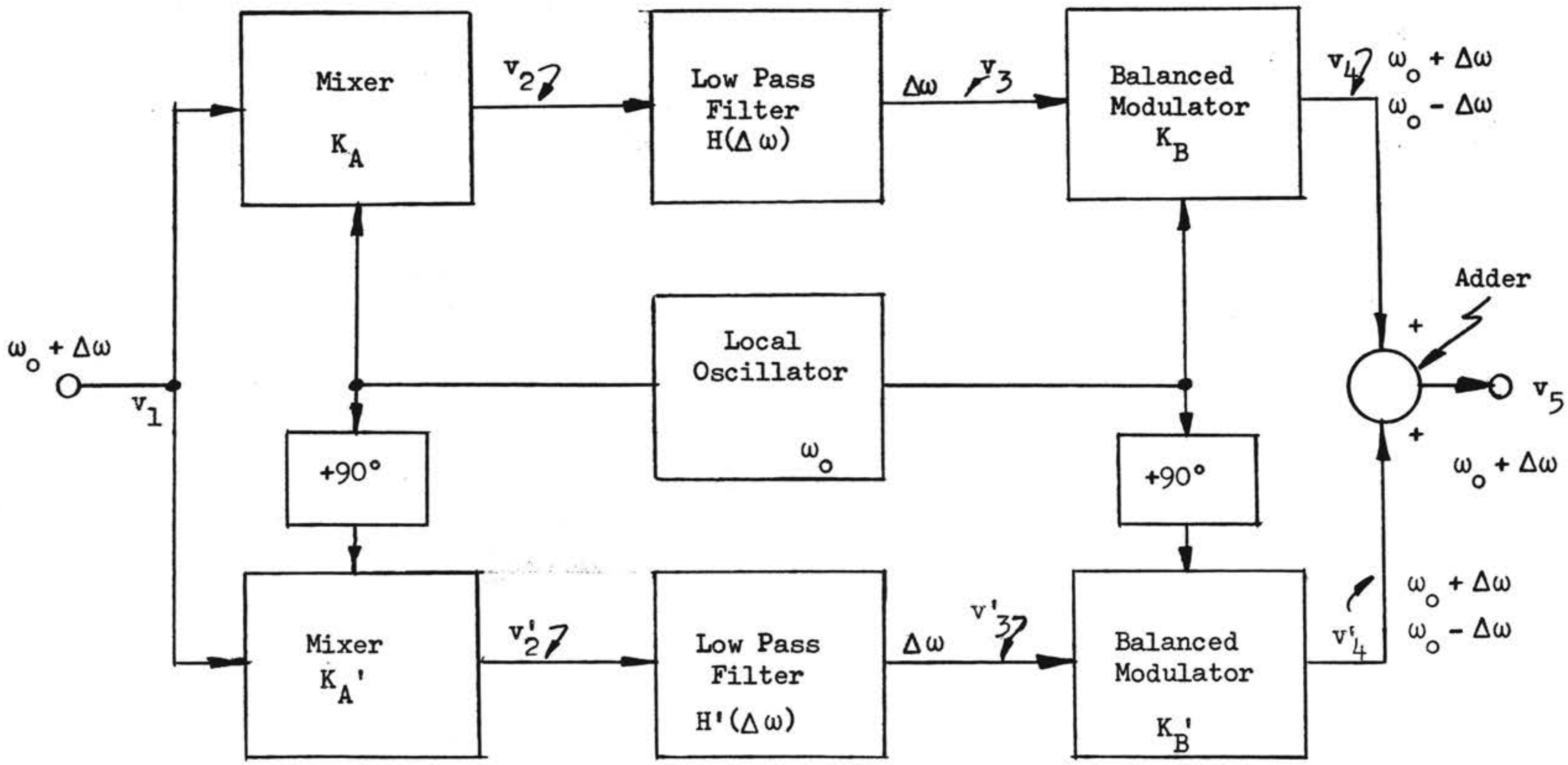


Figure 3-1. Block Diagram of Passband Generator

In operation, the incoming signal is mixed with the I and Q local oscillator signals in the appropriate mixers. A beat signal is obtained at each mixer output and is applied to the corresponding low-pass filter. Assume for the moment that each of these low-pass filters has an amplitude characteristic as shown in Figure 3-2 and that $\Delta\omega$ (the amount by which the input signal differs from the center frequency of the filter) is less than the cut off frequency of the low-pass filters, ω_c.

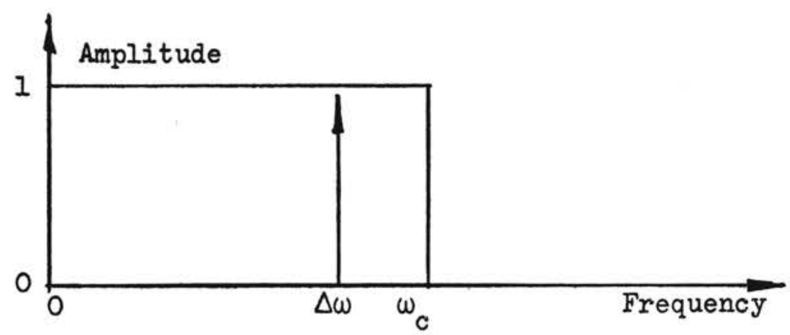


Figure 3-2. Low-Pass Filter Amplitude Characteristic

The output of the low-pass filters then consist solely of the unattenuated signals at $\Delta\omega$. These signals are identical in every respect except phase; the Q Channel lagging the I Channel signal by ninety degrees. The I and Q local oscillator signals are now modulated with their corresponding beat signals. Since the modulation process is balanced, no carrier component (ω_o) appears and the modulator outputs consist only of two sidebands, one at $\omega_o + \Delta\omega$ and one at $\omega_o - \Delta\omega$. Due to the two ninety degree phase shifts applied in the Q channel one set of sidebands, either the upper or lower, will be cancelled and the other reinforced when the modulator outputs are added. Which sideband is cancelled depends upon whether the input signal is above or below the center frequency of the filter. For example, if the applied signal is above the center frequency of the filter, then the two lower sidebands will cancel leaving only an upper sideband which has the same frequency as the input signal and which has been modified by the bandpass characteristic of the filter.

Should $\Delta\omega$ fall above ω_c (Figure 3-2), no beat signal would appear at the output of either the I or Q Channel low-pass filters and, hence, there would be no output from the Passband Generator. This establishes the bandwidth of the filter as twice the cut off frequency of either low-pass filter.

The operation of the filter may be better understood from the following analysis of its operation. Reference is made to the nomenclature of Figure 3-1 (page 30).

Consider the system described above with a signal $v_1 = V_1 \mathrm{Cos}(\omega_o + \Delta\omega)t$ applied to the input terminals where $(\omega_o + \Delta\omega)$ represents a signal above the center frequency of the filter (ω_o) by an amount $\Delta\omega$. This signal goes directly to the first mixer in each channel. Also applied to these mixers is a signal at ω_o generated by the local oscillator. The signal which feeds the mixer in the I Channel is given by $V_o \text{Cos}\ \omega_o t$ and the signal which feeds the Q Channel is given by $V_o \text{Cos}(\omega_o t + 90°)$.

Since these mixers are product devices, their outputs are given by

$$v_2 = K_A V_1 \text{Cos}(\omega_o + \Delta\omega)t \times V_o \text{Cos}\ \omega_o t \tag{3-1}$$

and

$$v_2' = K_A' V_1 \text{Cos}(\omega_o + \Delta\omega)t \times V_o \text{Cos}(\omega_o t + 90°). \tag{3-2}$$

Using the trigonometric identity

$$\text{CosACosB} = {}^1/2\,[\text{Cos}(A + B) + \text{Cos}(A - B)] \tag{3-3}$$

v_2 becomes

$$v_2 = \frac{K_A V_1 V_o}{2}\left[\text{Cos}(\omega_o + \Delta\omega + \omega_o)t + \text{Cos}(\omega_o + \Delta\omega - \omega_o)t\right]$$

$$v_2 = \frac{K_A V_1 V_o}{2}\left[\text{Cos}(2\omega_o + \Delta\omega)t + \text{Cos}(\Delta\omega)t\right] \tag{3-4}$$

and

$$v_2' = \frac{K_A' V_1 V_o}{2}\left[\text{Cos}(\omega_o t + \Delta\omega t + \omega_o t + 90°) + \text{Cos}(\omega_o t + \Delta\omega t - \omega_o t - 90°)\right]$$

$$v_2' = \frac{K_A' V_1 V_o}{2}\left[\text{Cos}(2\omega_o t + \Delta\omega t + 90°) + \text{Cos}(\Delta\omega t - 90°)\right]. \tag{3-5}$$

Each of these signals are fed to low-pass filters, the characteristics of which are assumed idential. Assume also that the filter characteristics are given by

$$H(\Delta\omega) = \begin{cases} F(\Delta\omega) & \Delta\omega \le \omega_c \\ 0 & \Delta\omega > \omega_c \end{cases}$$

where ω_c is the cut off frequency of the filter and that ω_c is less than $\omega_o/2$. If $\Delta\omega$ is less than ω_c, v_3 and v_3' are given by

$$v_3 = \frac{F(\Delta\omega)K_A V_1 V_o}{2} \mathrm{Cos}\Delta\omega t$$

and

$$v_3' = \frac{F'(\Delta\omega)K_A' V_1 V_o}{2} \mathrm{Cos}(\Delta\omega t - 90°) \ .$$

These two voltages represent the beat signal between the local oscillator and the signal applied to the input terminals of the filter. Their amplitudes are directly proportional to the characteristics of the low-pass filters (represented by $F(\Delta\omega)$) and it is these filters which determine the passband of the system.

The two voltages v_3 and v_3' are then each applied to the signal input terminals of a balanced modulator. The output of these devices consists solely of the product of the signals applied to each modulator, the carrier (ω_o) component having been balanced out.* Also fed to each modulator is a signal from the local oscillator. The I Channel modulator receives $V_o\mathrm{Cos}\omega_o t$ and the Q Channel modulator receives $V_o\mathrm{Cos}(\omega_o t + 90°)$; the same signals which supplied the first mixers. The outputs of the modulators v_4 and v_4' may then be written

*Assuming suitable filtering to remove undesired harmonics.

$$v_4 = \frac{F(\Delta\omega)K_A K_B V_1 V_o}{2} \text{Cos}\Delta\omega t \times V_o \text{Cos}\,\omega_o t$$

and

$$v_4' = \frac{F'(\Delta\omega)K_A' K_B' V_1 V_o}{2}\ \text{Cos}(\Delta\omega t - 90°)V_o\text{Cos}(\omega_o t + 90°)$$

where K_B and K_B' are the gain constants of the two modulators. Using the same trigonometric identity as applied to Equations 3-4 and 3-5, these may be rewritten

$$v_4 = \frac{F(\Delta\omega)K_A K_B V_1 V_o^2}{4}\left[\text{Cos}(\omega_o t + \Delta\omega t) + \text{Cos}(\omega_o t - \Delta\omega t)\right] \quad (3\text{-}6)$$

and

$$v_4' = \frac{F'(\Delta\omega)K_A' K_B' V_1 V_o^2}{4}\left[\text{Cos}(\omega_o t + \Delta\omega t) - \text{Cos}(\omega_o t - \Delta\omega t)\right]. \quad (3\text{-}7)$$

Let

$$M = \frac{F(\Delta\omega)K_A K_B}{4} \quad (3\text{-}8)$$

and

$$M' = \frac{F'(\Delta\omega)K_A' K_B'}{4} \quad (3\text{-}9)$$

then write

$$v_4 = MV_1V_o^2\left[\text{Cos}(\omega_o t + \Delta\omega t) + \text{Cos}(\omega_o t - \Delta\omega t)\right] \quad (3\text{-}10)$$

and

$$v_4' = M'V_1V_o^2\left[\text{Cos}(\omega\Delta t + \Delta\omega t) - \text{Cos}(\omega_o t - \Delta\omega t)\right]. \quad (3\text{-}11)$$

When these two signals are added together to form the output signal, it is found that

$$v_5 = v_4 + v_4' = V_1 V_o^2 (M + M') \mathrm{Cos}(\omega_o t + \Delta\omega t) + V_1 V_o^2 (M - M')$$

$$\mathrm{Cos}(\omega_o t - \Delta\omega t) \ . \tag{3-12}$$

If the two channels are made identical so that $M = M'$, then

$$v_5 = 2MV_1 V_o^2 \ \mathrm{Cos}(\omega_o t + \Delta\omega t) \tag{3-13}$$

which is a replica of the input signal modified by the constant M. Replacing M by its defined value yields

$$v_5 = \frac{F(\Delta\omega) K_A K_B V_1 V_o^2}{2} \ \mathrm{Cos}(\omega_o + \Delta\omega)t$$

or

$$G(\omega) = \frac{V_5}{V_1} = \frac{F(\Delta\omega) K_A K_B V_o^2}{2} = KF(\Delta\omega) \tag{3-14}$$

where $K = (K_A K_B V_o^2)/2$ and where $F(\Delta\omega)$ indicates that the amplitude of the transfer function depends upon the beat frequency $\Delta\omega$.

If the applied signal had been of the form $V_1 \mathrm{Cos}(\omega_o - \Delta\omega)t$, indicating that the input signal was below the local oscillator in frequency, a similar analysis would show that

$$G(\omega) = K\,F\,(\Delta\omega) \tag{3-15}$$

which is the same as Equation 3-14.

In effect, a passband centered at ω_o has been generated and the characteristics of this passband are directly determined by the characteristics of the low-pass filters used. The 3 decibel bandwidth of the resulting bandpass filter is exactly twice the 3 decibel bandwidth of either low-pass filter.

It can be seen that for complete cancellation of the unwanted sideband, the I and Q Channels must be identical in every respect. Any deviations from this requirement result in incomplete sideband cancellation, although possibly only under some circumstances. This requirement extends to both the shape and cut-off frequency of the two low-pass filters used. Consider the effects of two non-identical low-pass filters as demonstrated below.

Assume that the two low-pass filters have passband characteristics as shown in Figure 3-3 and that the two channels are otherwise identical. If a signal is applied to the Passband Generator which is less than 2 kilocyles from its center frequency (f_o), then all the conditions stated above for proper sideband cancellation are met and the passband generator output is a single sine wave at the same frequency as the input signal. If Δf is now increased to, say, 2.5 kilocycles, the I Channel passes the beat signal after attenuating it by an amount A_1. The Q Channel, however, passes its beat signal after attenuating it by an amount A_2. When the outputs of the balanced modulators are combined, incomplete cancellation of the unwanted sideband will result since the sideband output from each channel is of different amplitudes.

This condition may be corrected at 2.5 kilocycles by adjusting the gain of the two channels so that $A_1 = A_2$.

This leads to incomplete cancellation at other frequencies. It, therefore, appears that the two filters must be as nearly identical as it is possible to make them if maximum sideband suppression is to be obtained over the entire passband.

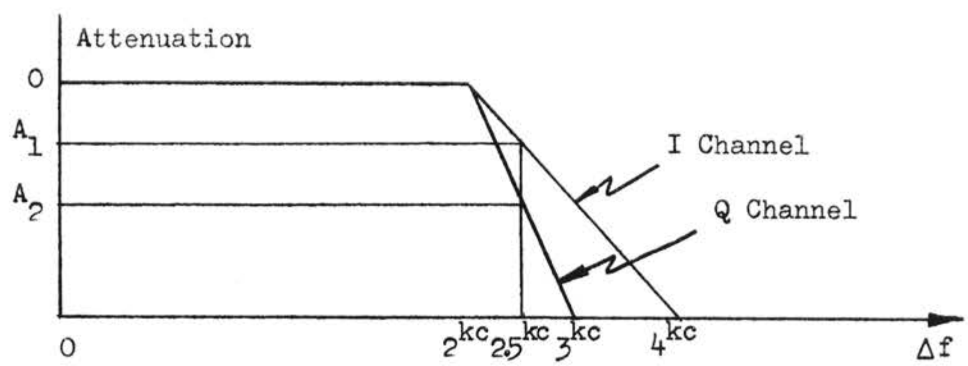


Figure 3-3. Two Non-Identical Low Pass Filters

Since the sideband suppression is directly related to the amplitude differences between the channels, it is a simple matter to calculate the required matching for a given amount of sideband suppression. If the two channels are matched within ten per cent (in amplitude) at every frequency in the passband, then the undesired sideband will be suppressed by 25.6 decibels with respect to the desired sideband. One per cent matching gives approximately 40 decibels, while obtaining 60 decibels suppression requires matching to within one-tenth of one per cent.

A similar calculation made on the tolerance for the ninety degree phase shifter reveals the following. Achieving 25 decibel sideband suppression requires a tolerance of ± 2.5 degrees. A suppression of 40 decibels requires ± 0.50 degrees. Appendix A contains examples of both of these calculations. It is apparent from these results that unless special care is taken in construction, about the best sideband suppression that can be expected is 30 decibels. The ninety degree tolerance should be less critical than the amplitude tolerance since it is possible to adjust this phase angle and since such an adjustment could easily be made as part of a set up procedure.

While the analysis above has been carried out assuming a single sine wave as the input signal a similar analysis may be carried out for an amplitude modulated signal. This shows that such a signal will be passed intact, providing, of course, that all sidebands fall within the passband of the filter. This analysis is presented in Appendix B.

CHAPTER IV

PASSBAND GENERATION USING MODULATION TECHNIQUES - II

The material in this chapter covers the circuitry and operation of the Passband Generator constructed by the author. Section 4.1 contains a circuit description and Sections 4.2 and 4.3 contain the results of measurements made on the generator. A setup procedure for the device is included in Appendix C.

4.1 Circuitry

The circuit presented below is not intended to represent any ultimate or final design for a filter of this type, but is rather a breadboard model constructed to investigate its principal of operation. There are a number of places where simplifications and improvements may be made to improve its performance.

Figures 4-1 and 4-2 show, respectively, the circuitry of the I (In-phase) and Q (Quadrature phase) channels of the generator. Since they are quite similar, they will be described together. Q channel transistor designations will be indicated by the appropriate nomenclature in parenthesis. For example, Q_1 (Q_3) refers simultaneously

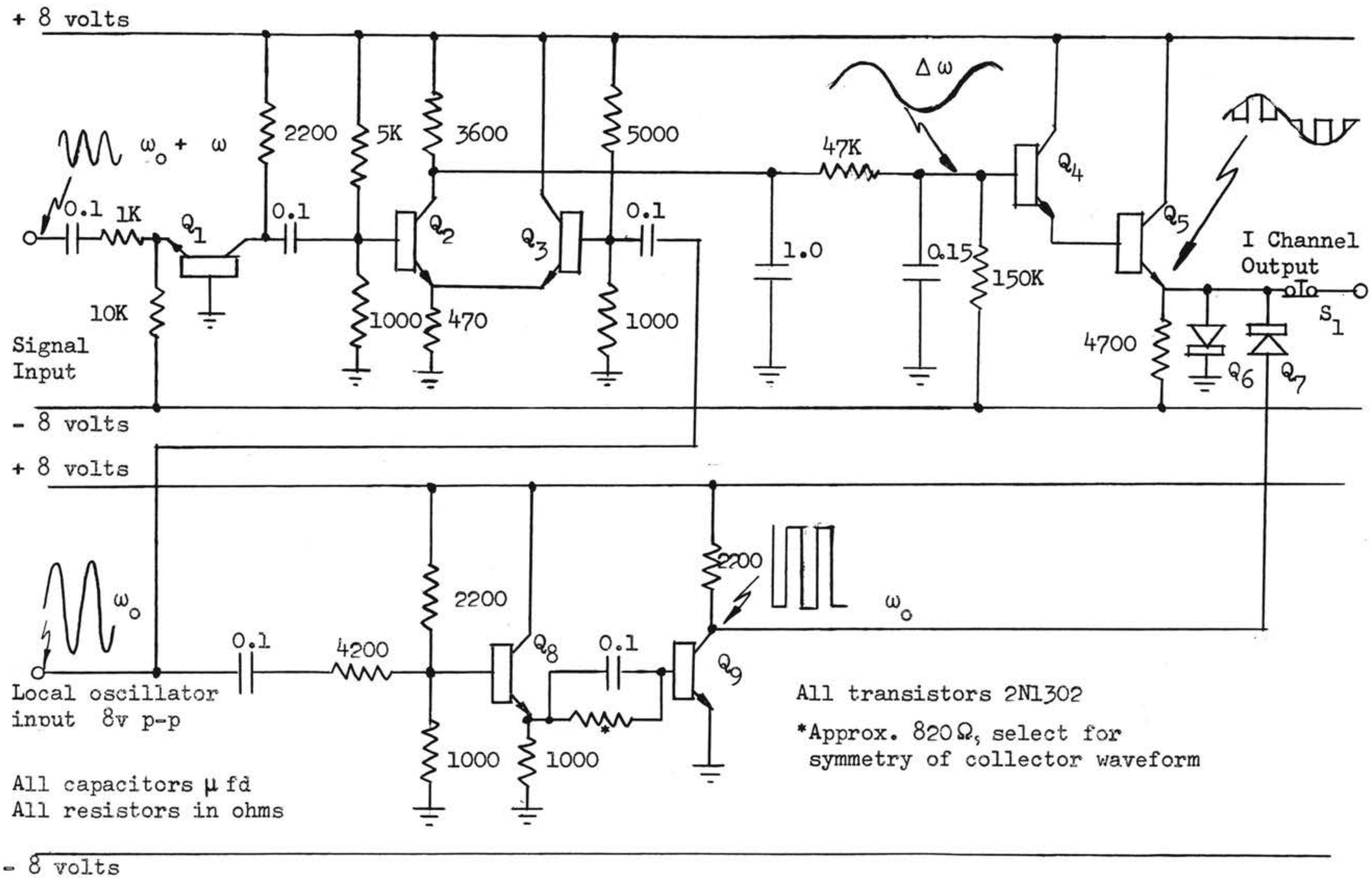


Figure 4-1. Schematic of Passband Generator I - Channel

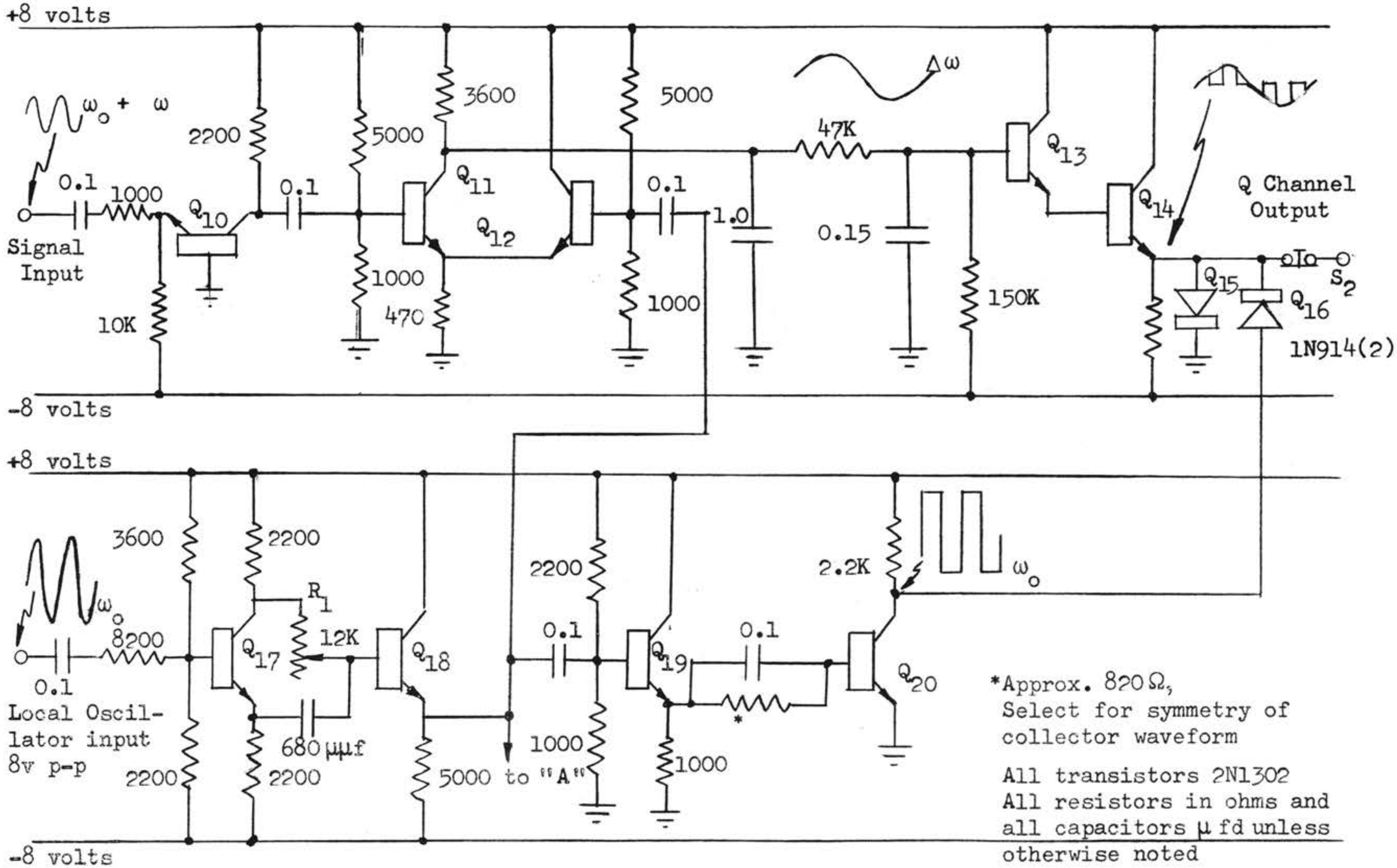


Figure 4-2. Schematic of Passband Generator Q - Channel

to I channel transistor Q_1 and Q channel transistor Q_3.

A signal fed to the input terminal of the filter goes directly to the emitters or Q_1 and Q_{10}. These two stages, operating in the common-base connection, provide isolation between the I and Q channels and prevent cross coupling of local oscillator signals fed back through the mixer stages Q_2 and Q_3 (Q_{11} and Q_{12}). The output of the common-base stage is fed to the base of Q_2 (Q_{11}) and the local oscillator signal fed to the emitter through a direct coupled emitter follower employed to isolate the mixer from the local oscillator input line. To insure effective mixer operation, a local oscillator signal of 8 volts (peak-to-peak) is employed. The output of the mixer, at the collector of Q_2 (Q_{11}) is fed to a two section low-pass filter consisting of the 3.6K collector load resistor, a 47K resistor and the 1.0 microfarad and 0.15 microfarad capacitors. These components determine the bandpass characteristics of the filter. Other low-pass filters may, of course, be inserted at this point. A Darlington emitter follower, Q_4 and Q_5 (Q_{13} and Q_{14}) is connected to the output of this filter to provide a high impedance load and a low driving impedance for the diode modulators Q_6 and Q_7 (Q_{15} and Q_{16}). The output of the low-pass filter is a signal whose frequency depends upon the difference in frequency between the local oscillator and input signals (called $\Delta\omega$ in Chapter II) and whose amplitude is dependent upon the transfer function of the low-pass filters employed.

Proper operation of the modulator stages requires that the emitter of $Q_5(Q_{14})$ be held at a low positive voltage to prevent conduction of Q_6 (Q_{15}) until a positive pulse is applied. Since all the diodes in the modulator are silicon types, a positive emitter voltage of 0.4 volts is used. The 150K resistor connecting the base of Q_5 (Q_{14}) to the minus 8 volt bus sets this voltage. A change in the low-pass filter configuration will probably make a change in the value of this resistance necessary.

The keying pulse for the modulators (which is a square wave switching between zero and plus 8 volts) is derived in the squaring circuit Q_9 (Q_{20}). Q_9 (Q_{20}) is driven from clamp to cutoff by the emitter follower Q_8 (Q_{19}) which is employed for isolation. The starred resistor in both Figures 4-1 and 4-2 must be selected for collector waveform symmetry because of individual vagaries in the transistors used.

To provide the ninety degrees phase shift required by the local oscillator signals used in the Q channel phase shifter, Q_{17} is used. The 12K potentiometer (R_1) provides the phase adjustment. Since this circuit must feed a high impedance load, emitter follower Q_{18} is directly coupled to the output of the shifter circuit. This follower feeds the mixer and modulator circuits in the Q channel.

It is a characteristic of this circuit that it has exactly ninety degrees phase shift at only one frequency. Every change in local oscillator frequency will, therefore, require a change in the setting of R_1. The author had no difficulty in achieving the required ninety degree phase shift from this circuit anywhere in the 25 kilocycle to 45 kilocycle range.

After modulation, there exists at the output of each channel an amplitude modulated signal (a carrier, two sidebands and harmonics of the same) with phase relationships as described in Chapter III. Since the harmonics generated in the modulation process are undesired, they must be filtered out someplace in the system. For simplicity, the author has chosen to use a single tuned circuit in the output stage for this purpose. The following sections of the Passband Generator then first cancel out the fundamental component of the square wave in each channel (the carrier of f_o) and then combine the two double sideband signals resulting to form (after filtering) the single output signal desired.

Two identical carrier subtraction circuits are used, one for each channel (Figure 4-3). No phase inversion is employed in these stages since 180 degrees shift was introduced in the carrier squaring circuits. To achieve carrier cancellation, it is necessary only to re-add the local oscillator signal (shifted ninety degrees for the Q channel) in the proper amount. Emitter followers Q_{24} and Q_{25} are employed for isolation purposes with adjustable phase lag circuits included to compensate for delays through the individual channels. Cancellation of the

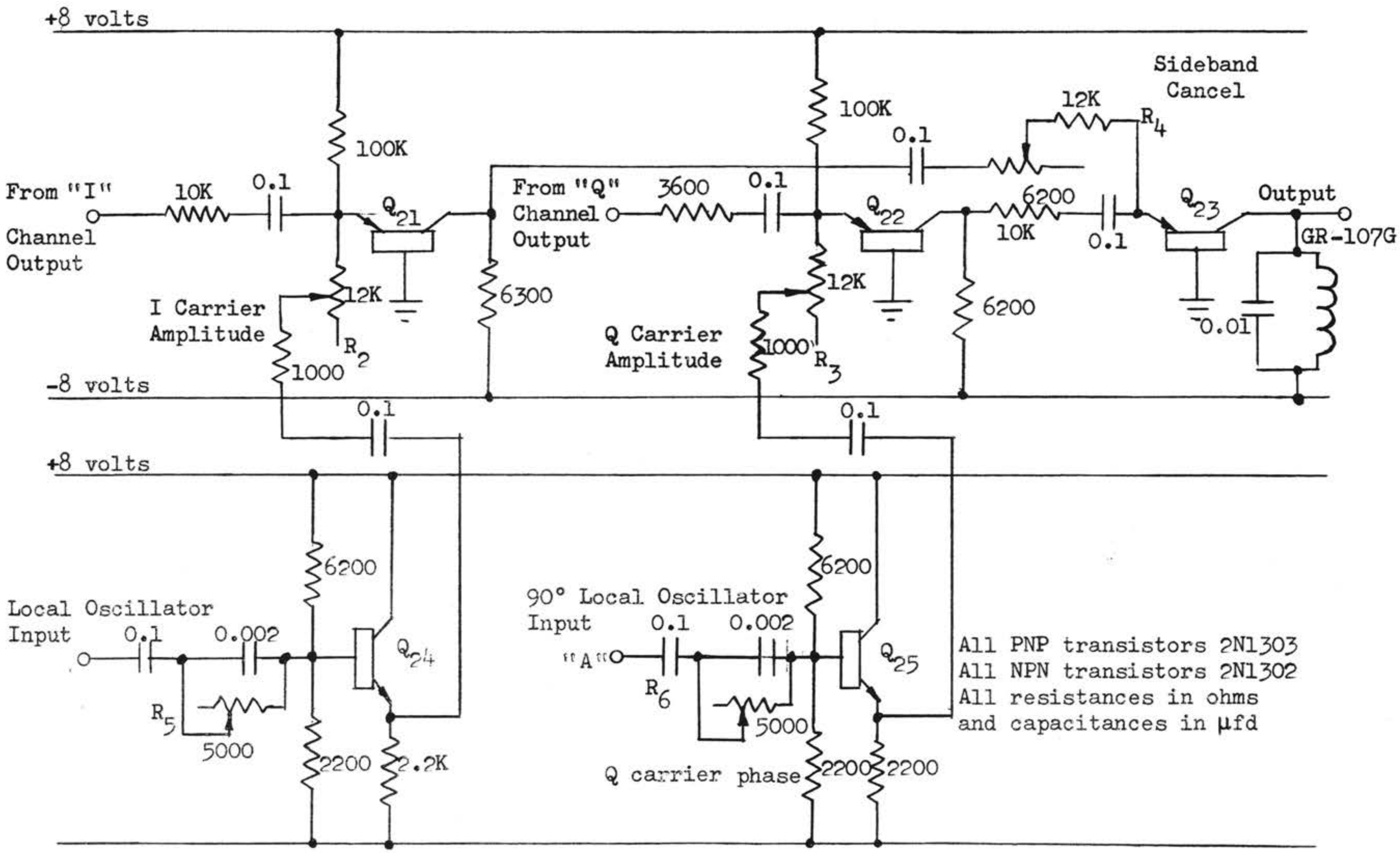


Figure 4-3. Schematic of Passband Generator Adder Stages

fundamental component of the carrier is achieved in common-base adders Q_{21} and Q_{22}. The 12K potentiometers R_2 and R_3 are used for carrier balance adjustments. Alternate adjustment of the corresponding phase and carrier level controls results in reducing the fundamental component of the carrier to below one millivolt. Carrier cancellation will be discussed more fully in the following sections.

The output of these adders is then fed to the final adder Q_{23} where the I and Q signals are combined. Cancellation of the undesired sideband is achieved by the adjustment of R_4 (in the output lead of Q_{21}) and R_1 the ninety degree phase shift control. The output of the Passband Generator appears at the collector of Q_{23} and is filtered to eliminate the undesired harmonics by the parallel tuned circuit shown (Figure 4-3). This tuned circuit in no way contributes to the selectivity properties of the device and other schemes of filtering may be employed.

An attempt was made to construct a balanced modulator which would operate effectively for modulating signals down to dc, but with little success. The primary problem was that of maintaining carrier balance under shifting dc conditions. This might be overcome with some sort of dc feedback arrangement, but the author felt the carrier cancellation technique would provide the desired results with the simplest circuitry and, hence, was used here.

4.2 Passband Characteristics

It is shown in Chapter III that the bandwidth of the Passband Generator is solely determined by the bandwidth of the two low-pass filters used and that it may be predicted to be twice the bandwidth of either low-pass filter. A check of Figures 4-1 and 4-2 will reveal that the low-pass filters used in this Generator are as shown in Figure 4-4 in equivalent circuit form. Z_i represents the input impedance of the Darlington emitter follower loading the filter and is approximately 100K. An asymptotic approximation to the actual amplitude characteristic of this low-pass filter may be calculated on the assumption that the second section does not load the first. Such a calculation reveals that each section has the same breakpoint frequency of 45 cps resulting in a high frequency asymptote slope of 40 decibel/decade starting at the breakpoint. This asymptote is plotted in Figure 4-5.

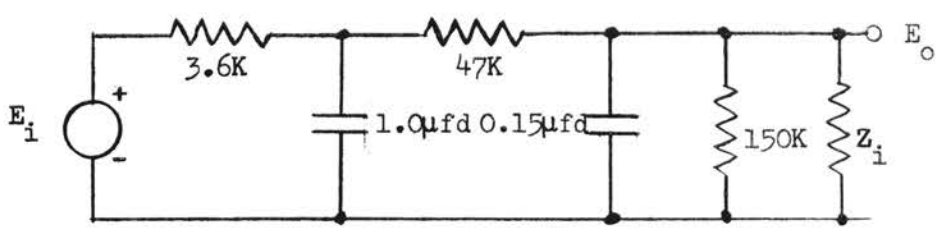


Figure 4-4. Equivalent Circuit of Passband Generator Low-Pass Filters

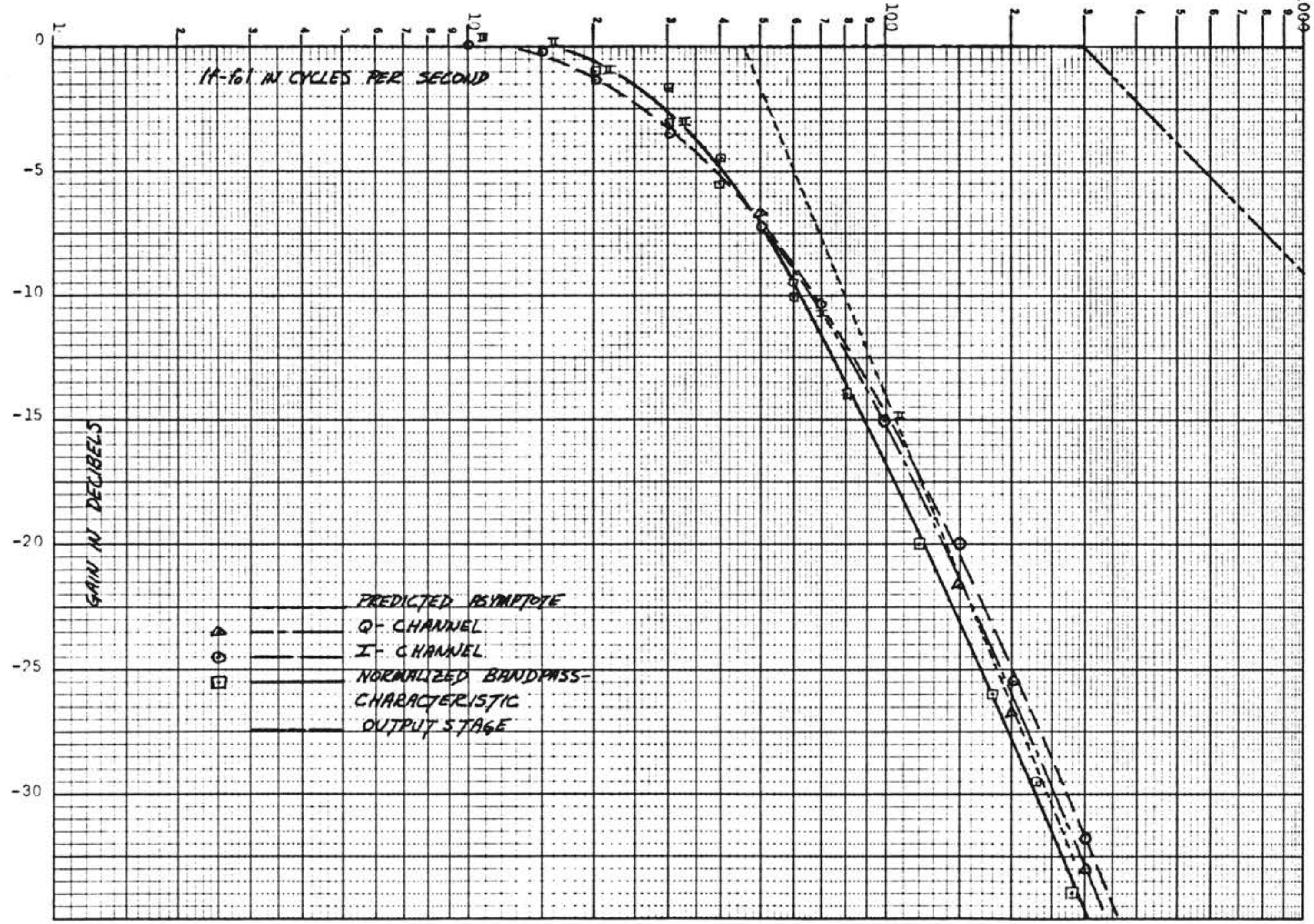


Figure 4-5. Measured and Predicted Amplitude Response for Passband Generator

The actual low-pass characteristic of each of the filters used was measured and plotted in the same figure. It may be seen that the two filters approach the asymptote very closely and are both 6 decibels down at the break-point frequency. The two low-pass filters were found to be everywhere within 1.5 decibels (corresponding to a voltage ratio of 1.18) of one another. The following section will consider this fact in relation to sideband suppression, but for the purposes of passband comparisons the two low-pass filters may be considered identical.

These measurements indicate that the Passband Generator should have a 6 decibel bandwidth of

$$2 \times 45 = 90 \text{ cps}$$

and the bandshape should correspond point-by-point to the shape of the filters shown in Figure 4-5. Bandpass measurements were made at five frequencies over the range of 25 to 45 kilocycles.* At each new center frequency, the filter was readjusted for both sideband and carrier cancellation before any measurements were made. The five resulting bandpass curves are shown in Figure 4-6. It must be emphasized that these five passbands do not exist simultaneously but are, rather, the result of the

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*A Hewlett-Packard (Palo Alto, Calif.) 402-A Wave Analyzer was used as a signal source. This analyzer has a linearly calibrated scale over the range of 20 cps to 50 kc so that highly precise determination of the signal frequency was possible.

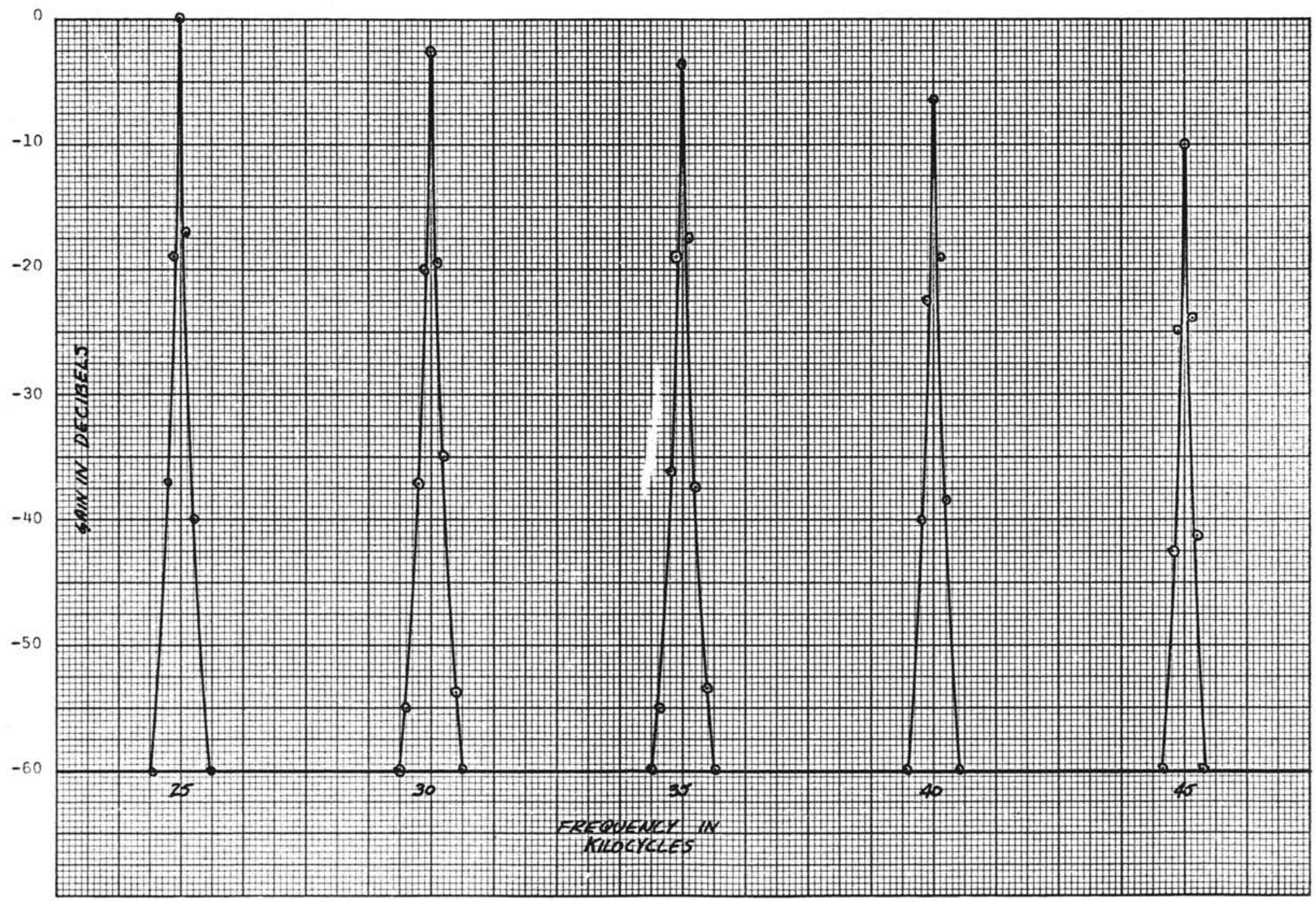


Figure 4-6. Measured Bandpass Characteristics at Five Different Center Frequencies

readjustment procedure mentioned above. Figure 4-6 also shows the variation of maximum gain with frequency which is believed due to high frequency roll-off in the transistors used. In order to facilitate a comparison between the measured and predicted bandpass characteristics, the bandpass data for the 40 kilocycle center frequency filter was normalized with respect to 40 kilocycles (f_o) and $|f - f_o|$ plotted against attenuation in Figure 4-5. It is apparent from this figure that the measured and predicted passband characteristics are in close agreement. It is also seen that the simple asymptotic approximation to the amplitude characteristic of the low-pass filters used provides a simple means for determining the passband characteristics of the resulting filter.

That this is so for all center frequencies tested may be implied from Figure 4-7 where the normalized passband characteristics for the Generator operating at the two extreme frequencies of 25 kilocycles and 45 kilocycles are plotted. It may be seen that these characteristics are essentially identical.

To insure that the passband of the parallel tuned circuit used on the output of the Passband Generator was sufficiently wide so as not to contribute to the over-all selectivity of the Generator, its selectivity curve was measured and is included in Figure 4-5 for comparison purposes.

With the selectivity characteristics of the Passband

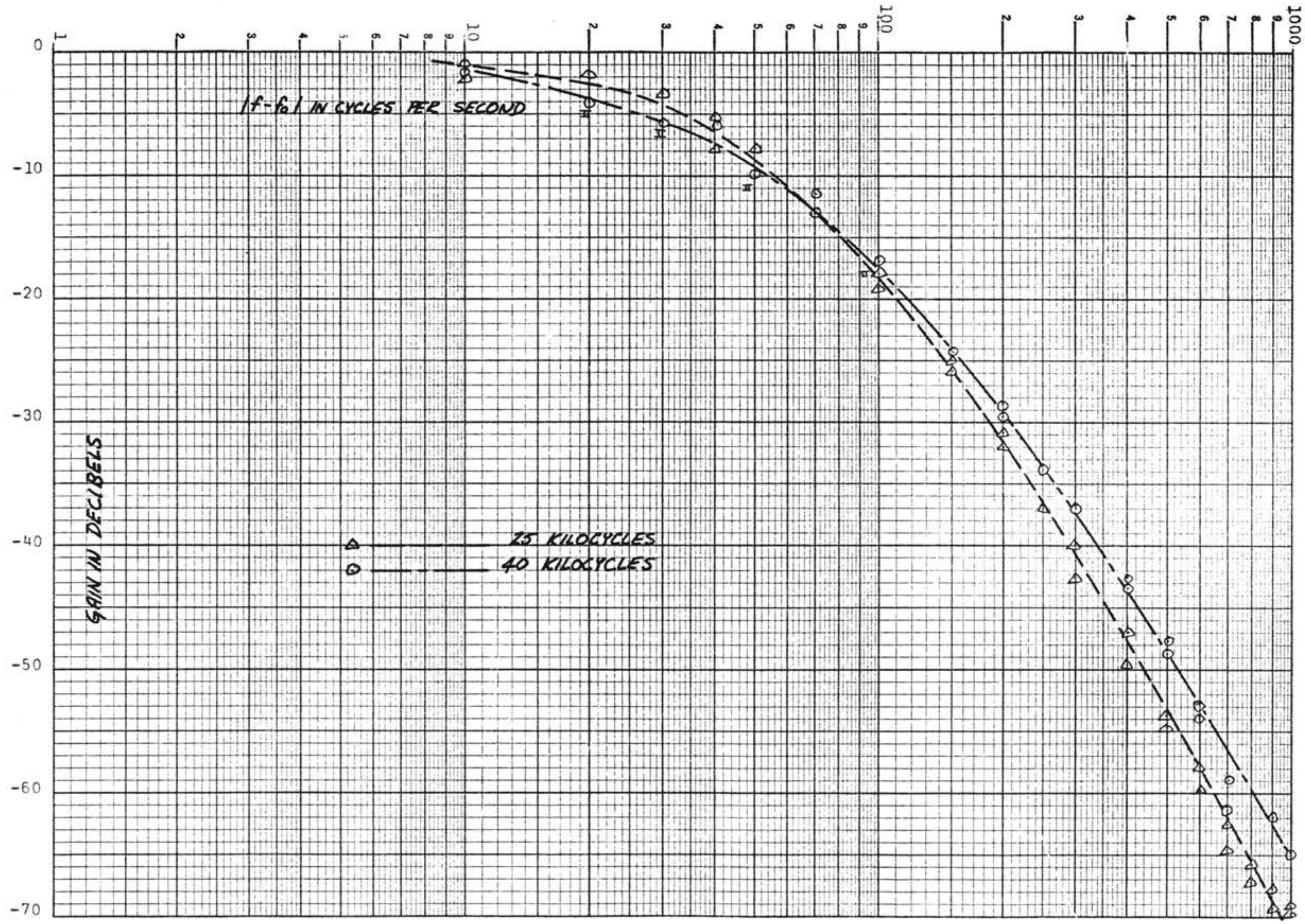


Figure 4-7. Comparison of Amplitude Response at 25 Kilocycles and 40 Kilocycles Center Frequencies

Generator known, it becomes possible to compute an effective Q for the device. The quality factor, Q, is defined to be the center frequency of the passband divided by the 3 decibel bandwidth.

$$Q = \frac{f_o}{BW_{3db}} . \qquad (4\text{-}1)$$

The 3 decibel bandwidth of the Generator may be read from Figure 4-5 to be 2 x 31 cycles per second. Therefore,

$$Q = \frac{40 \times 10^3}{2 \times 31} = 645. \qquad (4\text{-}2)$$

In an earlier model of the Passband Generator, the author attempted to achieve an even higher Q (than the above) by using two low-pass filters with breakpoint frequencies in the vicinity of 5 cycles per second. The equivalent circuit for the low-pass filters used is shown in Figure 4-8.

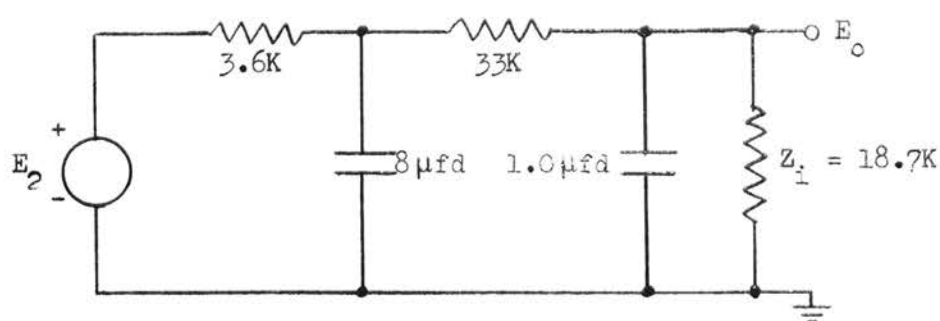


Figure 4-8. Narrow Band Two Section Low-Pass Filter

Once again using the assumption that the second filter section will not load the first, the two breakpoint frequencies of this circuit may be calculated. The first occurs at 5.51 cycles per second and the second at 8.5 cycles per second. In this case, the Z_i shown in Figure 4-8 represents the input impedance of a conventional emitter follower. The asymptotic approximation to the amplitude characteristic of these filters is shown in Figure 4-10.

The extremely narrow bandpass produced by these two low-pass filters made necessary the use of a statistical technique to determine the actual passband characteristic of the device. Even after one to two hours of warm-up, there was sufficient drift between the local oscillator and input signal generators to make straight-forward measurements impossible. The following procedure, then, was used to determine the passband characteristic of the device. After four hours of warm-up (for all equipment), four runs were made across the passband with the input signal generator-two in each direction. These measurements were made consecutively and all data points plotted in Figure 4-9. A curve representing the statistical average of these points was drawn and values taken from this curve for the normalized curve in Figure 4-10. Once again, it is seen that the measured values correspond closely to those predicted from the theory.

The 3 decibel bandwidth of this filter may be read

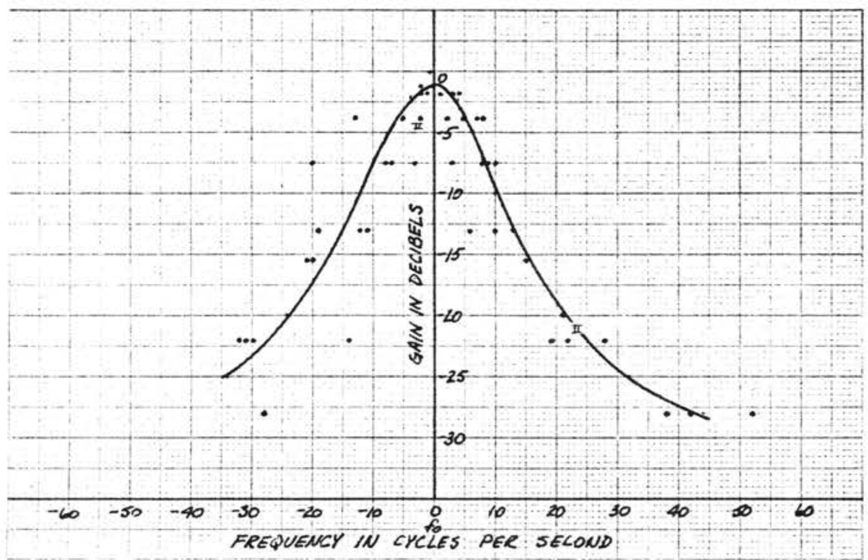


Figure 4-9. Statistical Determination of Amplitude Response

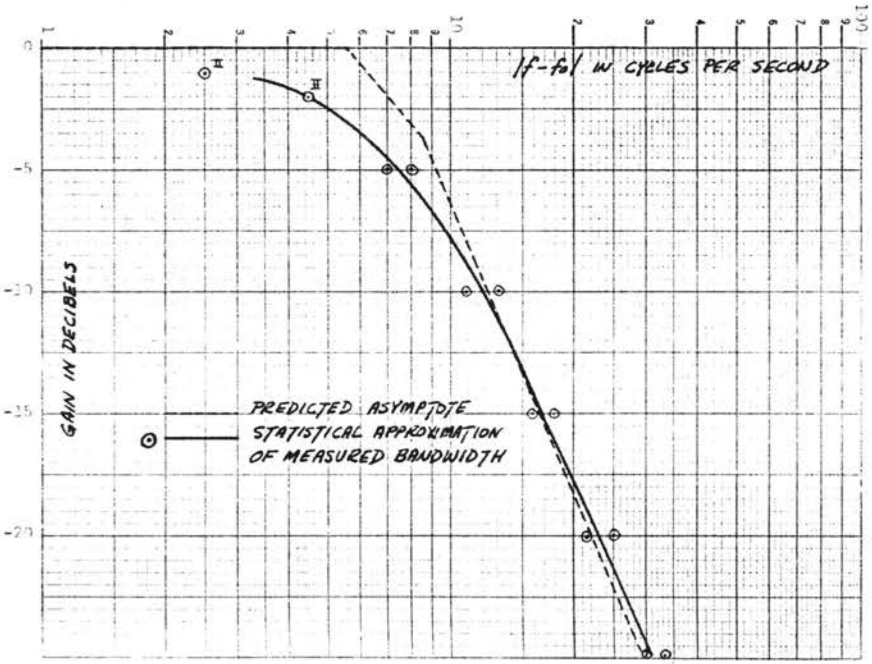


Figure 4-10. Comparison of Measured and Predicted Amplitude Responses

from Figure 4-10 to be $2 \times 5 = 10$ cycles per second. From this, and using Equation 4-1, the Q of the filter may be computed to be

$$Q = \frac{f_o}{BW_{3db}} = \frac{40 \times 10^3}{10} = 4000 \; . \qquad (4\text{-}3)$$

4.3 Gain and Noise Characteristics

4.3a Gain Characteristics

Ideally, any filter, regardless of type, should have a gain at each frequency within the passband which is independent of any external factors such as input signal level and environmental conditions. The introduction of active devices into the network, however, brings with it certain external limitations on the operation of the filter. The limitation to be considered here is the restriction placed on the range of allowable input levels.

The input mixers of the Passband Generator will operate properly only for a certain range of input signal levels. Outside this range, the mixer output either becomes distorted from overloading or excessively noisy from the use of too low a signal level. It has been experimentally determined that for this system the optimum range of input signal levels is 30 to 200 millivolts. Above this range, the beat signal from the mixers becomes distorted resulting in improper sideband canceling (to be discussed below) and below this range the output signal begins to be hidden in the system noise. Over most of this range of input signals, the system gain was found to be essentially constant and equal to one-third. Some roll-off was noted for high (above 100 millivolts) input levels, however. This is shown in Figure 4-11. It might be implied from this figure that the upper limit of input level should have been taken as 100 millivolts. Since system performance did not measurably suffer above this level (other than gain) there appeared to be no reason for not extending the input range to the 200 millivolt level previously mentioned.

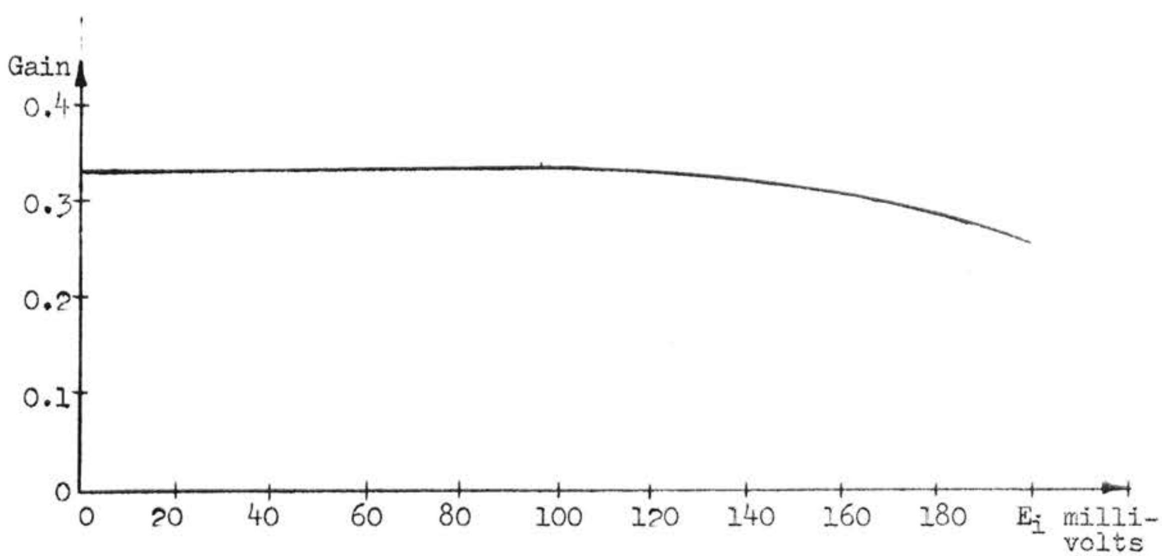


Figure 4-11. Passband Generator Gain as a Function of Input Level

4.3b Noise Characteristics

There are two main noise sources inherent in the operation of the Passband Generator. More properly, these noise sources should be considered as spurious output signals since they are the result of incomplete cancellation of signals produced within the Generator. It will be remembered from Chapter III that the desired output signal is produced by the addition of two double sideband signals. In the addition process, one sideband is reinforced and the other cancelled. Incomplete cancellation of the undesired sideband produces the first of the spurious outputs mentioned above. It will also be remembered that each of the above double sideband signals was produced by first generating an amplitude modulated signal and then subtracting the carrier from it. Incomplete cancellation of this carrier results in the second of the spurious output signals. These signals may be diagrammed as in Figure 4-12 where the input signal is assumed to be above the center frequency of the passband by Δf cycles per second.

The primary problem in the design of the Passband Generator is to achieve maximum cancellation of these spurious output signals. The factors affecting the cancellation of the unwanted sideband have been discussed in Chapter III and Appendix A and will be further considered below.

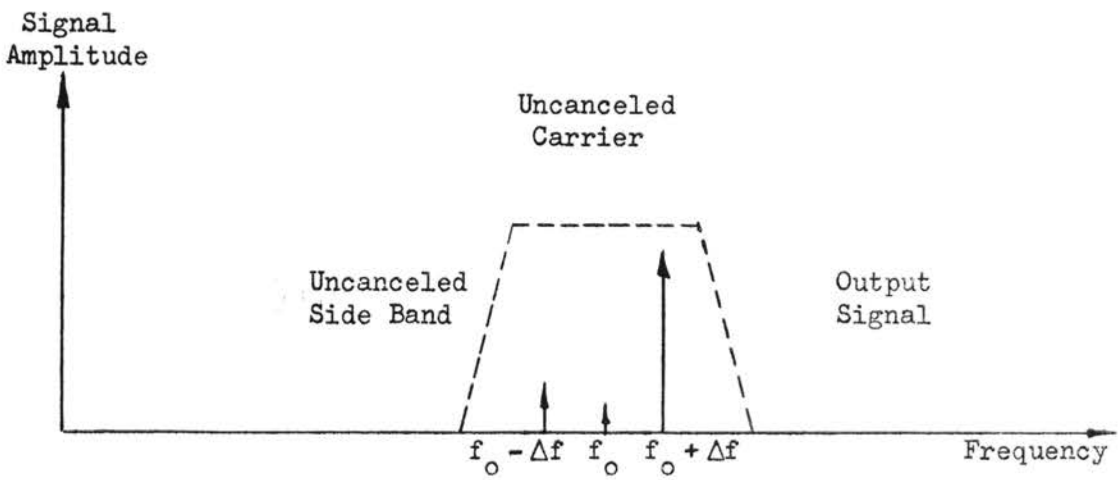


Figure 4-12. Output Signals of a Practical Passband Generator

Carrier cancellation is achieved by removing the fundamental component of the amplitude modulated square wave available at the output of the modulators. While this process should theoretically be perfect (with complete cancellation resulting), the practical limit is reached at about 1 millivolt rms where stray noise pick-up becomes apparent. This level corresponds to approximately 30 decibels below a standard output level of 40 millivolts rms (150 millivolt rms being taken as the standard test signal input level).

In the circuit constructed by the author, it was consistently possible to reduce the carrier to below one millivolt rms. With care taken in layout and design, there appears to be no reason why this cannot be reduced to any further desired level. In any subtraction situation such as this, the purity of the subtracting sine wave is of utmost importance. For this reason, careful attention must be given to the linearity of the stages handling the subtracting signal. Any non-linearity introduced by these stages will be reflected as less than maximum carrier cancellation.

Strictly speaking, carrier cancellation was found to be maximum only at the center frequency and input signal level for which the Passband Generator was set-up. Within the range of allowable input levels (25 to 200 millivolts), the variation of carrier level was found to be 3 millivolts maximum. That is, if the carrier level was initially reduced to its one millivolt minimum varying the input signal over the range of 25 to 200 millivolts caused the carrier level to increase to approximately 4 millivolts at the extremes.

This problem arises because it is necessary to direct couple through both channels of the filter. A change in input level causes a corresponding change in the dc output level at the input mixers. This dc level shift is carried to the modulators where it appears as a shift in carrier amplitude. This in turn causes the unbalance in the carrier output level mentioned above. The solution to this problem is to keep the input signal level small enough so that changes in level do not cause measurable dc shifts at the mixer outputs. This has the added advantage of insuring the purity of the mixer output signal which is a necessity if the undesired sideband is to be fully suppressed.

The variation in carrier level with changes in the center frequency (local oscillator frequency) of the filter was found to be as shown in Figure 4-13. No attempt was made to readjust the carrier cancellation controls for this measurement.

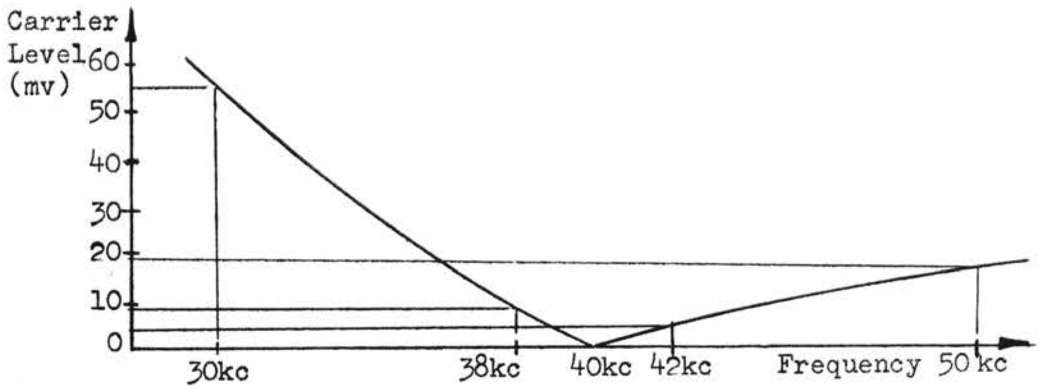


Figure 4-13. Variation of Carrier Level With Local Oscillator Frequency

Carrier cancellation is seen to be a maximum only at the chosen center frequency of the Passband Generator (40 kilocycles in this case). These previous two measurements seem to indicate that this device is limited to single center frequency, single input level signals if optimum performance is to be obtained.

As may be seen from Figure 4-5, the I and Q channel low-pass filter characteristics were found to be everywhere within 1.4 decibels corresponding to a voltage ratio or 1.18:1. This says that at their worst, the two channels are within 18% of each other. Referring to Equation C-2 reveals that a sideband suppression of

$$20 \log \left(\frac{1+k}{1-k}\right) = 20 \log \frac{1+0.85}{1-0.85} = 20 \log 12.3 = 21.8 \text{ db.}$$

21.8 decibels minimum may be expected where

$$k = \frac{G_2}{G_1} = \frac{1.00}{1.18} = 0.846$$

and assuming that the ninety degree phase shifter is set at exactly ninety degrees. Higher attenuations will be obtained elsewhere in the passband. Verification of this calculation proved to be difficult due to the erratic nature of the stray noise picked up by the circuitry. It was possible to measure the relative amplitudes of the components of the output spectrum at a few points in the passband, however. These measurements revealed that the carrier level was consistently 30 decibels below the desired output signal and the undesired sideband was within the range of 20 to 25 decibels below this same reference everywhere in the passband.

The main problem in achieving accurate sideband cancellation over the entire passband appears to be the degree of channel matching which may be achieved. Lack of accurate matching introduces a beat component into the output signal, the amplitude of which depends upon the degree of matching achieved. In this Passband Generator, the beat component appeared on the output signal as about 15% modulation at the beat frequency. This beat was predominant as the beat frequency approached zero.

CHAPTER V

A TWO PATH DIGITAL FILTER - I

(ANALYSIS)

As was mentioned in Chapter I, the similarity between the operation of the Digital Filter and the Passband Generator leads one to the conclusion that some combination of these devices might be possible. It is the purpose of this chapter to investigate one possible approach to such a combination device. The chapter following will discuss the circuitry and operation of a filter of this type constructed by the author.

The circuitry for such a combination device might take the form of Figure 5-1. Two identical low-pass paths are used with a pair of gating transistors included in each path. These gating transistors are operated much in the manner of the clamp transistors used in the simple four-path digital filter (Figure 2-3). The switching waveforms are symmetrical in both time and amplitude so that one transistor in each path is always saturated. Double sideband signals are generated at the output of each low-pass path by this switching action. These signals are then added to obtain the desired output signal. This additive process will be seen to be the same as the process

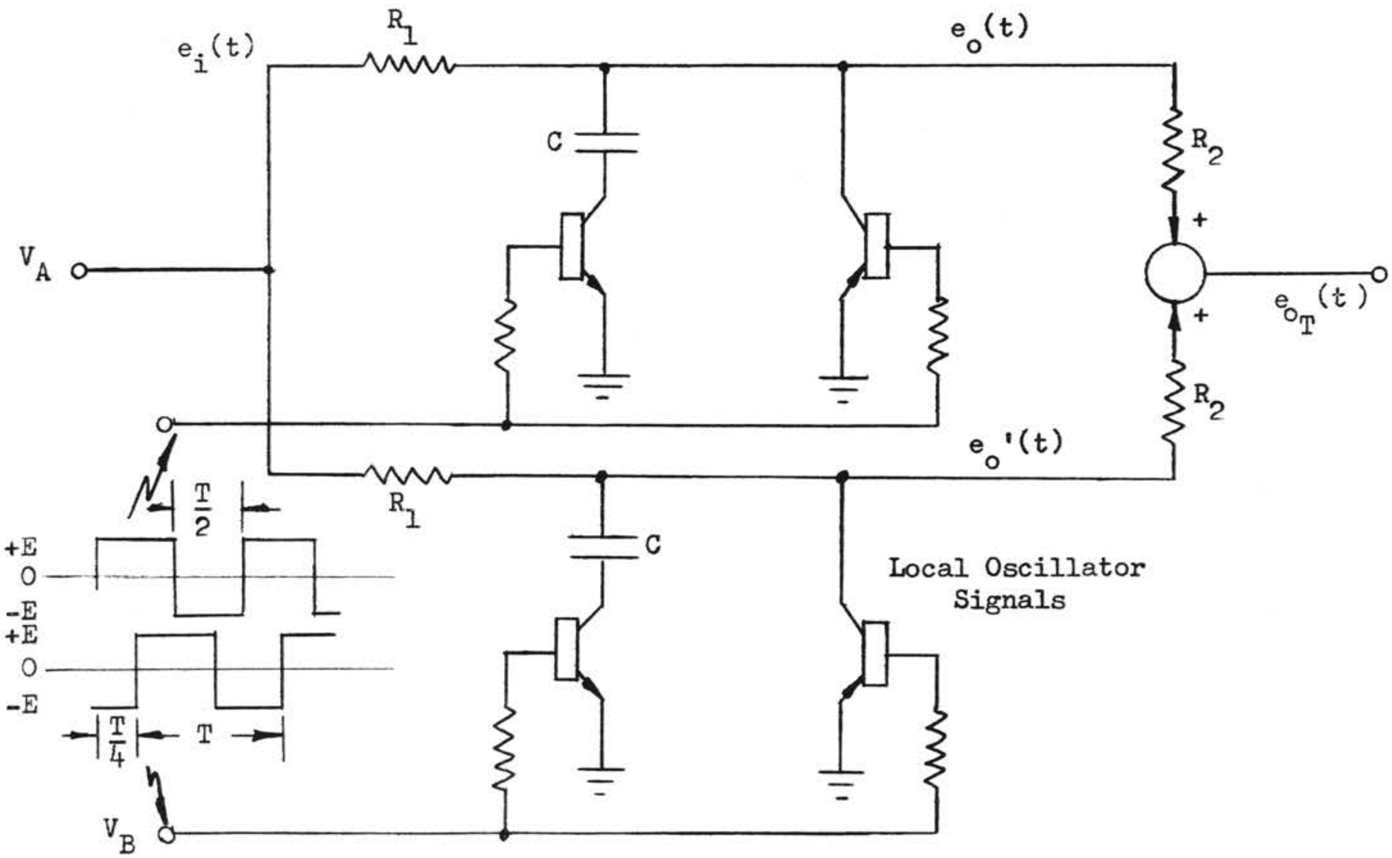


Figure 5-1. Basic Two-Path Digital Filter

used in the output stage of the Passband Generator.

The circuit of Figure 5-1 may be analyzed as follows. Consider each path separately for the moment (see Figure 5-2a). The two transistor switches may be represented by the SPDT switch as indicated. This switch is assumed to oscillate between its two positions at ω_o with zero transition time. An input signal at a frequency slightly above ω_o (by $\Delta\omega$) is applied to the input.

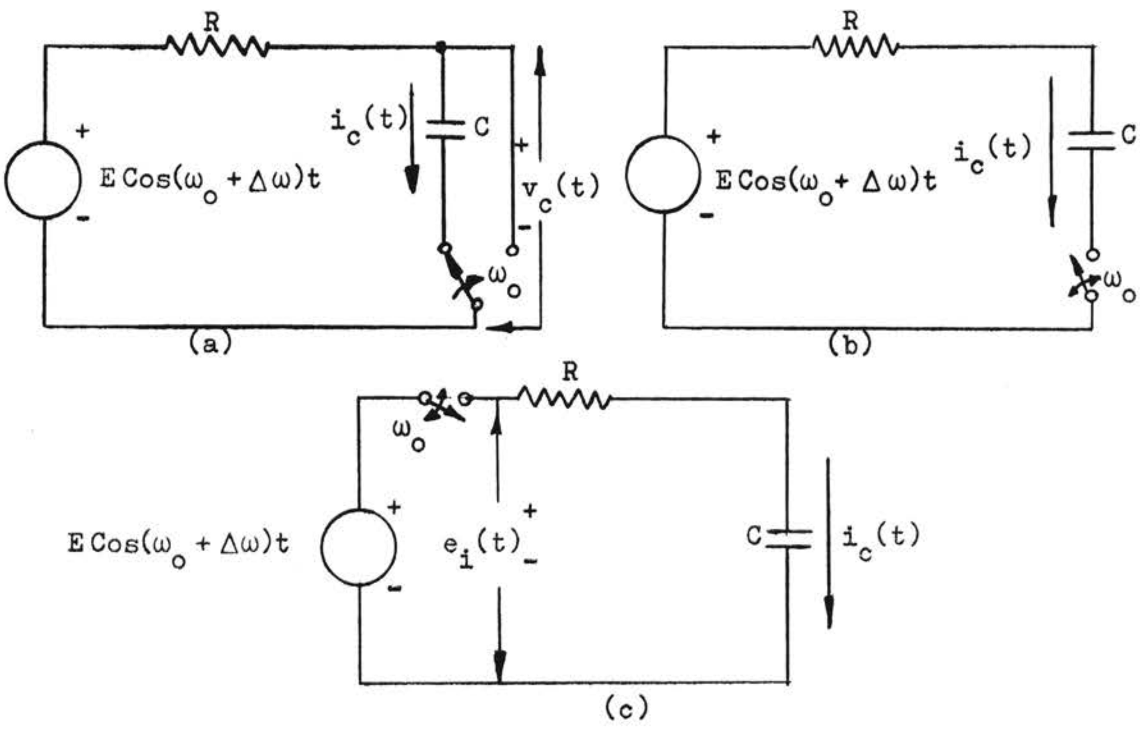


Figure 5-2. (a) One Path of the Two-Path Filter
(b) First Simplification for Analysis
(c) Second Simplification for Analysis

Analysis will proceed as follows. An expression for $i_c(t)$ will be obtained and from it $v_c(t)$ developed. The effect of the switching operation on $v_c(t)$ will then be considered and an output expression developed from it.

The presence of the right-hand switch position (where the capacitor is shorted out) will have no effect on $i_c(t)$ so it may be eliminated from the circuit for this analysis (Figure 5-2b). Since this is a simple series circuit, the position of the switch in the circuit has no effect on its operation and the switch may be moved to the input as shown in Figure 5-2c. It is, therefore, possible to compute the capacitor current on the basis of a sampled voltage input. The voltage at the input of the RC series circuit ($e_1(t)$) is then given as the product of $e_i(t)$ and a sampling function S(t).

$$e_1(t) = e_i(t) \times S(t)$$

$$e_1(t) = E \operatorname{Cos} (\omega_o + \Delta\omega)t \times \sum_{n=1}^{\infty} a_1 \operatorname{Cos} n\, \omega_o\, t \qquad n = 1, 3, 5 \ldots \tag{5-1}$$

For simplicity, only the fundamental component of the sampling function will be considered.*

$$e_1(t) = E \operatorname{Cos} (\omega_o + \Delta\,\omega)t \times a_1 \operatorname{Cos} \omega_o t$$

*Higher order harmonics are filtered out in the output stage.

or

$$e_1(t) = \frac{Ea_1}{2}[\text{Cos}\,\Delta\omega t + \text{Cos}\,(2\omega_o + \Delta\omega)t]\,. \qquad (5\text{-}2)$$

The input signal may, therefore, be represented as the superposition of two linear sources as shown in Figure 5-3.

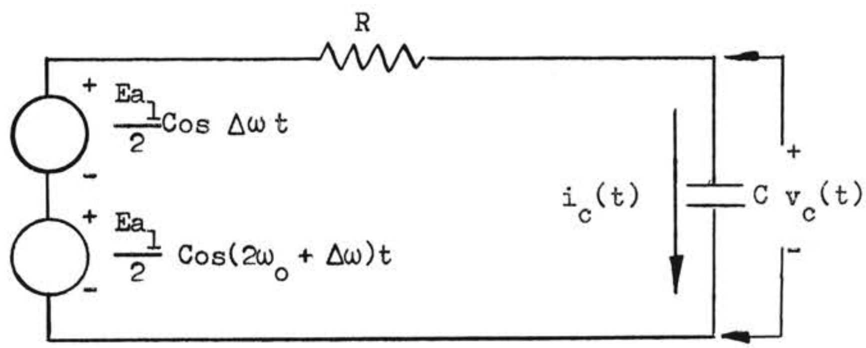


Figure 5-3. Representation of Input by Two Linear Sources

By superposition,

$$I_c(\omega) = \frac{Ea_1}{2\sqrt{2}} \times \left[\frac{1}{R - j\,\frac{1}{\Delta\omega C}}\right] + \frac{Ea_1}{2\sqrt{2}} \left[\frac{1}{R - j\frac{1}{(2\omega_o + \Delta\omega)\,C}}\right].$$

(5-3)

The voltage across the capacitor is then given by

$$E_c(\omega) = I_c(\omega) \times X_c(\omega)$$

or

$$E_c(\omega) = \frac{Ea_1}{2\sqrt{2}}\left[\frac{1}{R - j\frac{1}{\Delta\omega C}}\right]\left[-j\frac{1}{\Delta\omega C}\right] + \frac{Ea_1}{2\sqrt{2}}\left[\frac{1}{R - j\frac{1}{(2\omega_o+\Delta\omega)C}}\right]\left[-j\frac{1}{(2\omega_o+\Delta\omega)C}\right] \tag{5-4}$$

Expanding this expression yields

$$E_c(\omega) = \frac{Ea_1}{2\sqrt{2}}\left[\frac{1}{j\Delta\omega RC - 1}\right] + \frac{Ea_1}{2\sqrt{2}}\left[\frac{1}{j(2\omega_o + \Delta\omega)RC - 1}\right]$$

Since $\omega_o >> \Delta\omega$

$$E_c(\omega) = \frac{Ea_1}{2\sqrt{2}}\left[\frac{1}{j\Delta\omega RC - 1}\right] \tag{5-5}$$

or

$$e_c(t) = \frac{Ea_1}{2}\left[\frac{1}{j\Delta\omega RC - 1}\right] \text{Cos}\,\Delta\omega t\ . \tag{5-6}$$

The voltage across the capacitor is seen to consist primarily of a beat signal (at frequency $\Delta\omega$) whose amplitude is determined by the characteristics of the low-pass filter used. The action of the right-hand switch position (shorting position) will now be considered.

The voltage across the capacitor is now seen to be sampled by the shorting switch (Figure 5-4). This sampling process may be represented by

$$e_o(t) = e_c(t) \times S(t)$$

or

$$e_o(t) = \frac{a_1 E}{2}\left[\frac{1}{j\Delta\omega RC - 1}\right] \text{Cos}\,\Delta\omega t \times \sum_{n=1}^{\infty} a_n \text{Cos}\, n\omega_o t$$

$$n = 1, 3, 5, \text{---} \tag{5-7}$$

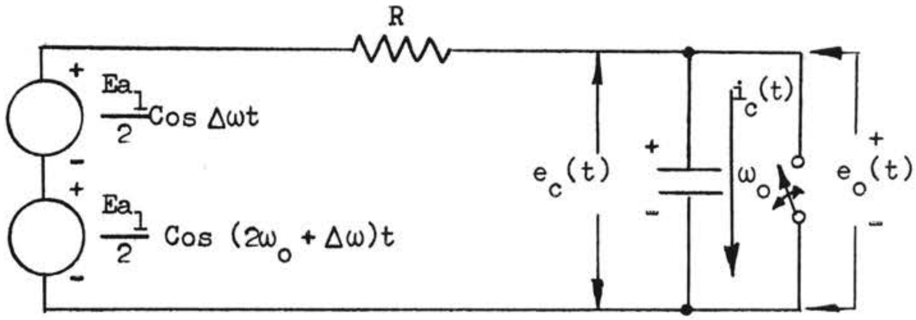


Figure 5-4. Action of Shorting Switch

Again, only the fundamental component of the sampling function need be considered. Therefore,

$$e_o(t) = \frac{a_1 E}{2}\left[\frac{1}{j\Delta\omega RC - 1}\right] \text{Cos}\,\Delta\ t \times a_1 \text{Cos}\ \omega_o t \qquad (5\text{-}8)$$

$$e_o(t) = \frac{a_1^2 E}{4}\left[\frac{1}{j\Delta\omega RC - 1}\right]\left[\text{Cos}(\omega_o + \Delta\omega)t + \text{Cos}(\omega_o - \Delta\omega)t\right] \qquad (5\text{-}9)$$

which is the desired double sideband output.

A similar analysis carried out for a sampling function ninety degrees out of phase with the one used above yields

$$e_o'(t) = \frac{a_1^2 E}{4}\left[\frac{1}{j\Delta\omega RC - 1}\right]\left[\text{Cos}(\omega_o + \Delta\omega)t - \text{Cos}(\omega_o - \Delta\omega)t\right] \qquad (5\text{-}10)$$

where a sampling function of the form

$$S(t) = \sum_{n=1}^{\infty} a_n \text{Cos}\,(n\omega_o t + 90°) \qquad n = 1, 3, 5 ---$$

is assumed. The total output of the filter is then the sum of $e_o(t)$ and $e_o'(t)$ or

$$e_{o_T}(t) = e_o(t) + e_o'(t)$$

$$= \frac{a_1^2 E}{2}\left[\frac{1}{j\Delta\omega RC - 1}\right]\left[\mathrm{Cos}\,(\omega_o + \Delta\omega)t\right] \qquad (5\text{-}11)$$

which is the desired output signal. As in the case of the Passband Generator, the output signal is seen to be a reproduction of the input signal modified in amplitude by a term depending on $\Delta\omega$. Although it will not be explicitly demonstrated here, the analysis of circuit tolerances carried out for the Passband Generator in Appendix A will also apply here. This is true since the same process is used to arrive at the output signal in each case.

CHAPTER VI

THE TWO-PATH DIGITAL FILTER - II

6.1 Circuitry

The circuitry for the two-path digital filter constructed by the author is shown in Figure 6-1. Switching waveforms V_A and V_B were obtained from the carrier squaring circuits described in Chapter IV (Figures 4-1 and 4-2) at the collectors of Q_9 and Q_{20}, respectively.

A signal applied to the input terminal is fed directly to the low-pass paths discussed in the previous chapter. Switching is accomplished in the four transistors Q_A, Q_B, Q_C, and Q_D. A symmetrical switching signal (in both amplitude and time), in conjunction with a symmetrical switching transistor configuration (PNP-NPN combinations) form the simple, yet effective, gating system. Differences in transistor saturation voltages are compensated for by the carrier balance controls. The two double sideband output signals are combined in common-base adder Q_E. Output filtering is achieved by means of a tuned circuit in the collector of this same transistor. While a center frequency of 40 kilocycles was chosen, any other frequency in the range of 25 to 50 kilocycles could have been used.

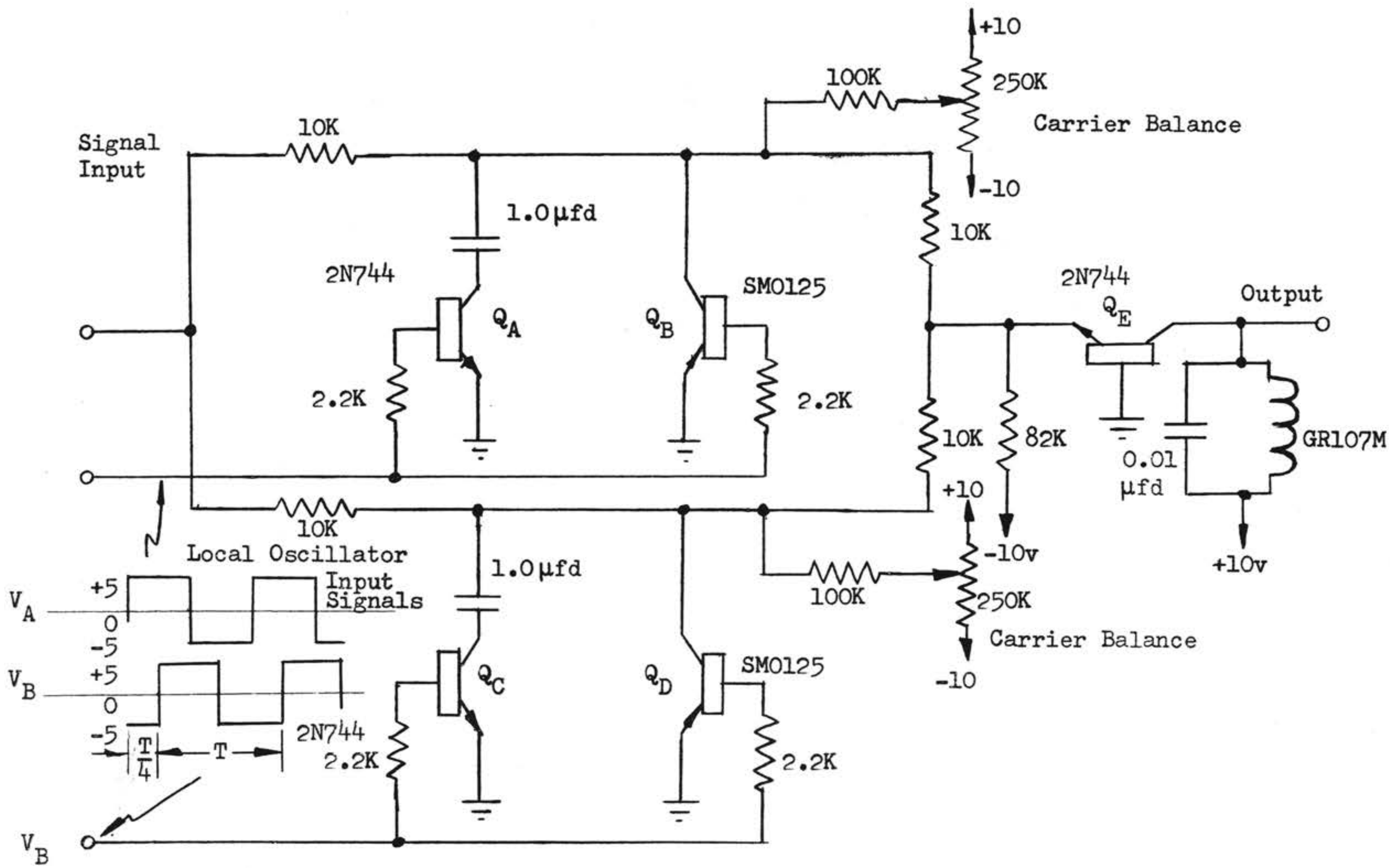


Figure 6-1. Schematic of Two-Path Digital Filter

Since this range was determined by the tuning range of the output filter, the substitution of a different filter at this point would allow the center frequency to be selected at will.

6.2 Gain and Noise Measurements

The results obtained with this circuit were as expected from the analysis of Chapter V. A passband was generated with a center frequency of 40 kilocycles and its bandshape was a duplicate of the characteristics of the low-pass filters used. The equivalent circuit shown in Figure 6-2 was used to calculate the transfer function of the low-pass filters. The transfer admittance for this filter may be computed to be

$$\frac{I_o(\omega)}{E_2(\omega)} = \frac{R}{2 + j\omega RC} .$$

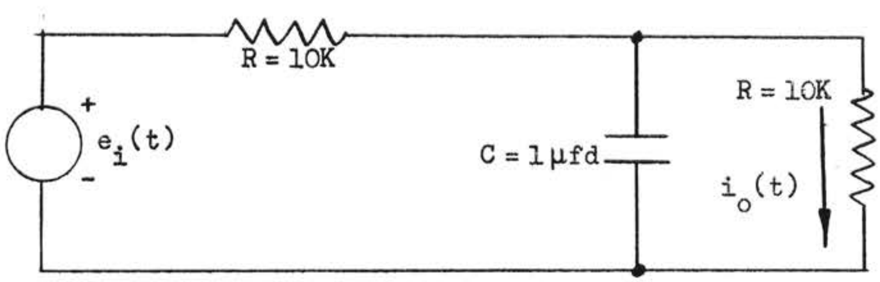


Figure 6-2. Low-Pass Filter Circuitry

Such a transfer function exhibits a breakpoint in its amplitude-frequency characteristic at

$$|\omega RC| = 2$$

or

$$f = \frac{1}{\pi RC} .$$

For R = 10K and C = 1.0 μfd, f = 31.9 cps. The expected bandwidth of the resulting passband is then 2 x 31.9 or 63.8 cps. This corresponds to a Q of 615. This amplitude-frequency characteristic (in asymptotic form) is plotted in Figure 6-3 as is the measured characteristic of the bandpass filter. The passband characteristic has been normalized with respect to the center frequency of the filter.

Since the same carrier generating circuits were used here as in the Passband Generator (Chapter IV) this filter exhibits much the same carrier noise characteristics as that device. One important difference noted here, however, was that the carrier level could be consistently reduced to a level six to ten decibels less than that attainable in the Passband Generator. This improvement may be attributed to the self-balancing nature of the circuitry used here. The carrier subtraction circuits of the Passband Generator were the primary cause of noise in that device.

A second trouble spot in the Passband Generator that was eliminated here was the input mixers. These mixers limited the range of input signals to a relatively small

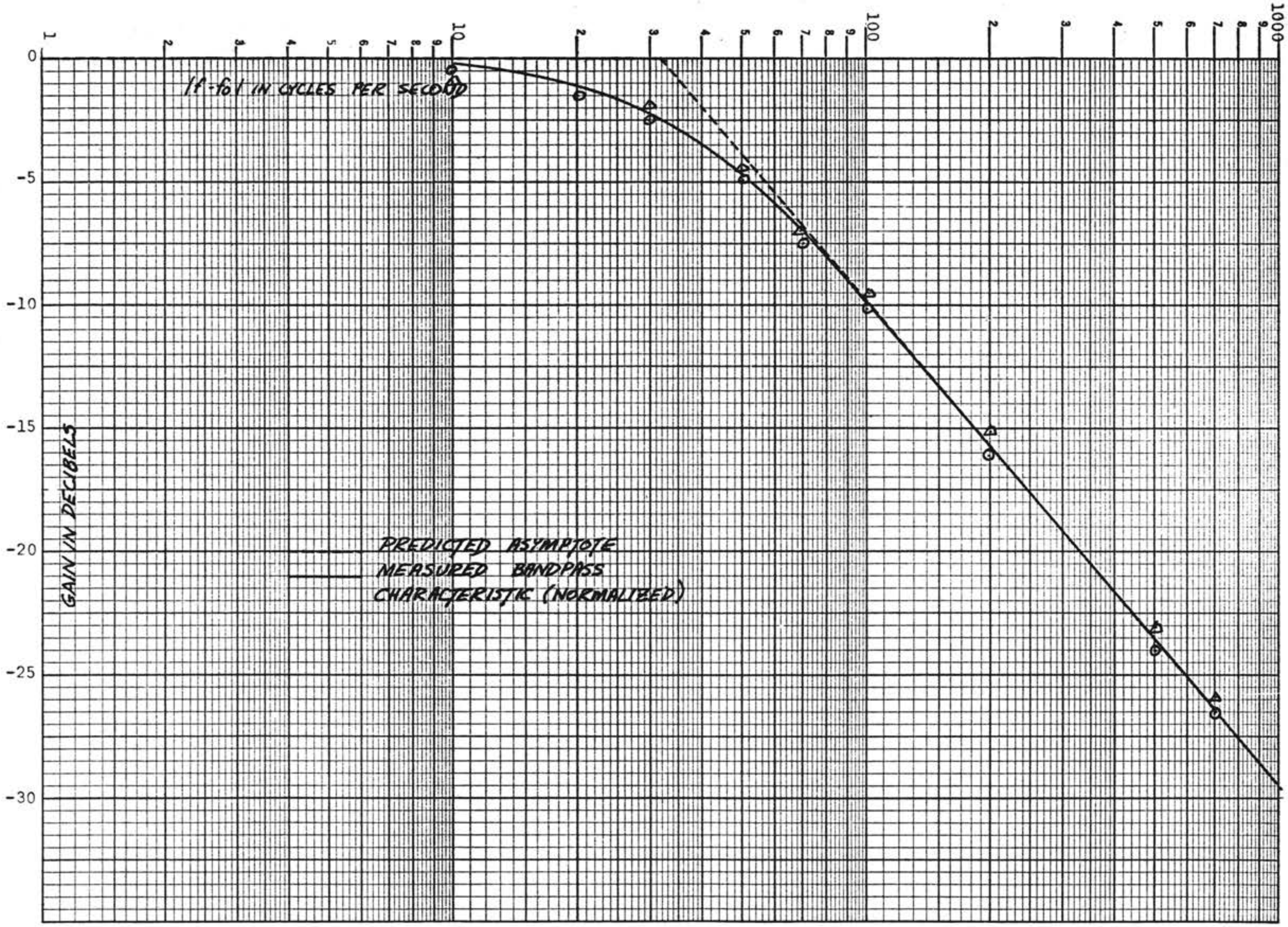


Figure 6-3. Comparison of Measured and Predicted Amplitude Response Characteristics

range. The author has successfully operated this device over the range of input levels from 25 millivolts to 1.5 volts rms. The upper limit determined here was the maximum level to which the collector-base junction of the switching transistors may be forward biased. It was also found that no readjustment of the carrier balance of phase shift controls was required with changes in input level. This is in direct contrast to the operation of the Pass-band Generator where each change in input level required a corresponding change in carrier level and phase. It must be said here, however, that a finite length of time was required for the filter to "settle down" after each change in level. The filter output during this transient period was characterized by incomplete carrier suppression. Given sufficient time, however, the carrier suppression returned to its previously established minimum with no further filter adjustments. The length of this settling period depended upon the magnitude of the input voltage change and was about five seconds for a change from two volts to 0.5 volts.

In an attempt to gain maximum sideband suppression over the entire passband, the filter components (the two 10K resistors and the 1.0 microfarad capacitor) were chosen to be within one per cent of their corresponding component in the opposite path. This led to a maximum ripple on the output signal of five to ten per cent which contrasts favorably with the ten to fifteen per cent obtained in the Passband Generator. A further reduction of this ripple component in the output would require channel matching to less than one per cent.* Since both of these requirements may be carried further than achieved here, there appears to be no reason why one per cent ripple should not be attainable.

A two-path digital filter may take other forms than used here. One other possible configuration is shown in Figure 6-4. This particular configuration appears simpler than the device discussed above, but the non-symmetrical switching signals used add a hidden complexity which may introduce difficulties not experienced with the circuit of Figure 6-1.

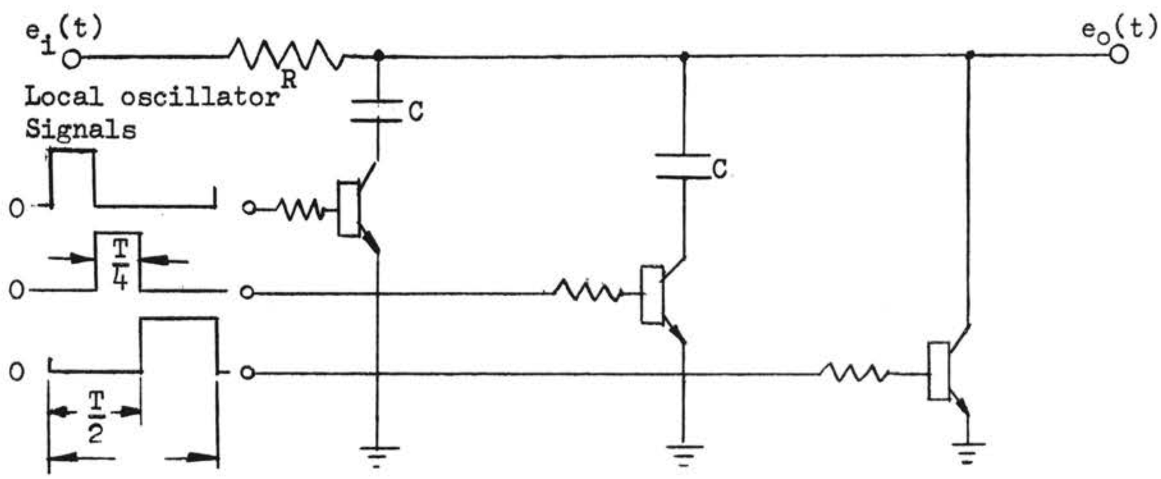
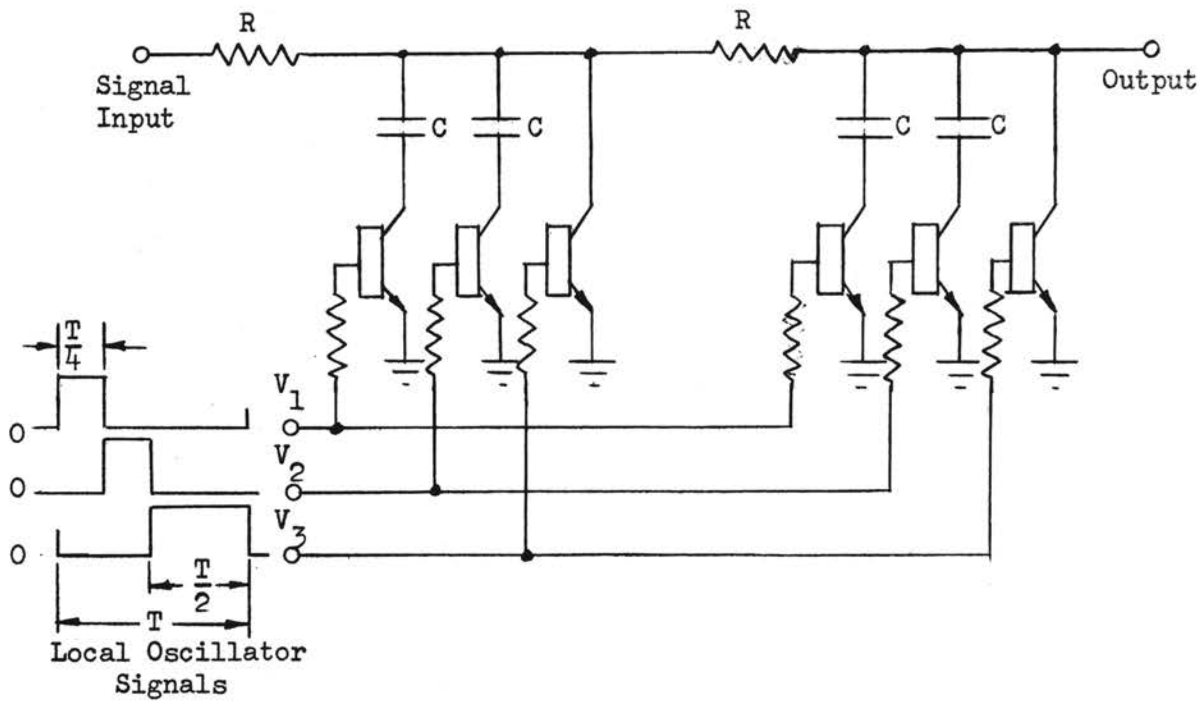


Figure 6-4. Alternate Two-Path Digital Filter

*This is to say that the two input resistors should be matched to less than one per cent, the two capacitors to less than one per cent, and so forth, regardless of their nominal values.

If one has a four-path digital filter already in operation, the above two-path device may be implemented by just shorting out two adjacent (in time) capacitors. This will also introduce a change in bandwidth, however.

If it is desired to use more than one low-pass section in the filter, the circuits may be arranged as shown in Figure 6-5. It can be seen that in a multi-section filter a considerable savings in the number of transistors required may be realized through the use of a two-path filter. A further savings may be realized in that only two low-pass paths must be matched here as against four paths in the digital filter. This could easily mean considerable savings in component costs with the high degree of matching required by these devices.



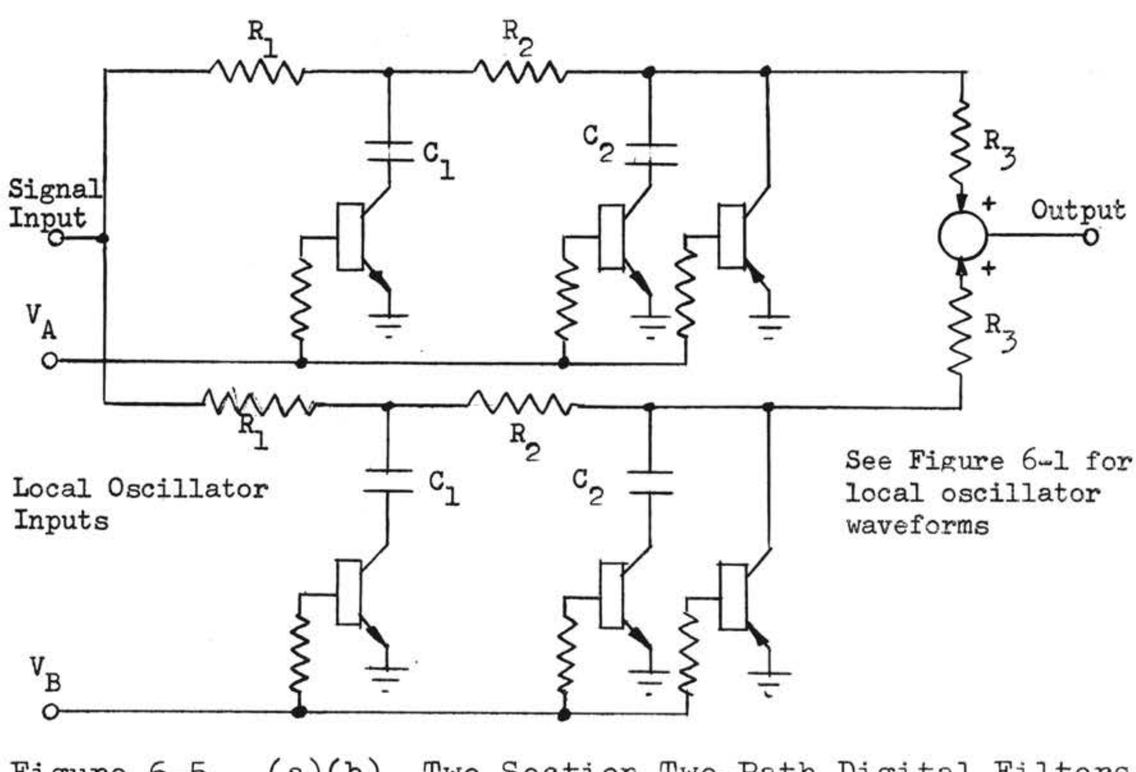


Figure 6-5. (a)(b) Two Section Two-Path Digital Filters

CHAPTER VII

SUMMARY AND CONCLUSIONS

Although the text of this thesis has covered three devices, really only one subject has been discussed. The central theme throughout has been an investigation of the various devices that may be used in the design of a specialized type of bandpass filter. The prime advantage of filters of this kind is that they have independent center frequency and bandwidth thus making the realization of unusually high circuit Q's possible without the use of specialized, and often costly, or impossible components.

The passband characteristics of the filter are determined entirely by a number of low-pass filters, thus bringing into practice the low-pass bandpass analogy often used in circuit theory. The only restrictions placed on these filters is that they must have an absolute cut-off frequency less than one-half the center frequency of the passband to be generated and that their amplitude characteristics be as nearly identical as it is possible to make them (Appendix A). This insures maximum cancellation of the spurious output signals generated in the filter.

In connection with all of these filters, it was verified that the resulting passband could be predicted from a knowledge of the characteristics of the low-pass filters incorporated in the circuit. In fact, it was seen that the passband characteristic of the Generator was simply the low-pass characteristic and its "mirror image" centered about the center frequency of the filter as shown in Figure 7-1.

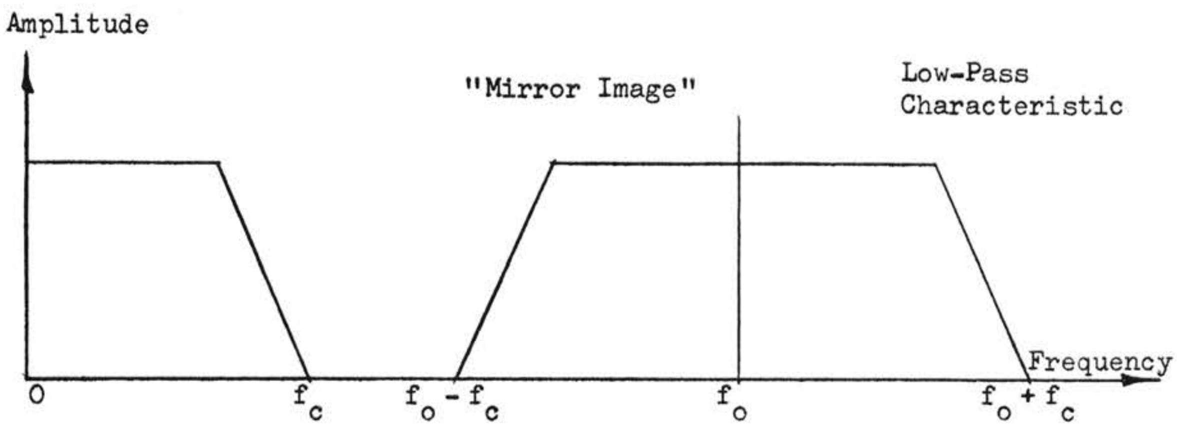


Figure 7-1. Determination of Passband Characteristics

It was also demonstrated that the center frequency of the filter could be selected at will and independently of the selection of the low-pass filters.

The Digital Filter was the first filter of this kind to be investigated. Chapter II consists of a short review of its theory of operation and a summary of the measurements made on the device. A filter using quadrature modulating functions (the "Passband Generator") was next investigated. A discussion and analysis of the operation of the device was presented in Chapter III. The chapter following (Chapter IV) contains a detailed circuit description of the unit constructed by the author along with data on its operating characteristics. An analysis and description of a Two-Path Digital Filter was next presented (Chapter V). This last device may be considered as a combination of the Digital Filter and the Passband Generator in the sense that while its configuration corresponds closely to the latter its switching mode is primarily that used in the former.

The direct correspondence between the operation of the Two-Path Digital Filter and the Passband Generator is apparent from the analyses presented in Chapters III and V. Both analyses conclude with the addition of a pair of double sideband signals to produce the output signal. The correspondence between these two devices and the Digital Filter may be implied from the analysis of the Two-Path filter. It was seen in this analysis that each path produced a double sideband signal as its output. Since the simple Digital Filter consists of four such paths, the output of this device should be four double sideband signals added together. Since the switching signals in the Digital Filter are displaced from one another by ninety degrees, it can be seen that the appropriate sidebands will be canceled in the output resulting (theoretically) in the single desired output signal.

Due to the common basis of operation shared by these devices, it is possible to draw a number of conclusions concerning them as a group from knowledge gained from them individually. First, and possibly most important, is the matching requirement placed on the low-pass filters used. Although Barber (6) and Tucker (9) have pointed out this requirement in connection with the filter using quadrature modulating functions apparently no consideration has been given it in connection with the Digital Filter. It is apparent from the comments made above that the channel matching requirement is every bit as critical in the Digital Filter as in the other devices discussed. The phase tolerance computed in connection with the Passband Generator, while not as critical as the channel matching tolerance, also applies directly to the digital filter. This tolerance carries over as a tolerance on the pulse timings in the digital device.

Other measured characteristics of the Passband Generator will not apply directly because they are a function of the circuitry used and not basic to the system. One difficulty involved in connection with this device is worthy of further discussion here, however. This is the difficulty involved in connection with direct coupling all the way through the device. Temperature stability problems are introduced by this requirement as well as the steady state level shift problem associated with changing input levels. Tucker (9) avoided this difficulty entirely by transformer coupling his balanced modulators to the low-pass filters in each path and accepting the resultant notch in the center of the passband. The author avoided the notch by the direct coupling technique mentioned above, but it is a matter of debate as to whether the added complexity is worth the more sophisticated result or not.

The Two-Path Digital Filter discussed in Chapters V and VI overcomes many of the problems associated with the Passband Generator while achieving identical results. Its primary advantages lie in its simplicity and the ease with which it may be adjusted for operation. This simplicity appears as fewer components required than in the Passband Generator and in the use of symmetrical switching waveforms as against the four one-fourth duty cycle pulses required for the Digital Filter. This simplicity also manifests itself in less carrier noise than in either of the other two devices and in the lack of one operational control ("sideband cancellation") over the Passband Generator. The automatic carrier rebalancing feature of the Two-Path filter with changes in input level is a further advantage of this device. The Passband Generator is totally lacking in this area and is one of the primary reasons for its lack of practicality.

It has been the intention of the author to conduct this investigation described here with an eye towards the possible application of these devices as selectivity determining elements in communications receivers. Unfortunately, it is impossible at this point to draw any real conclusion concerning the practicality of these devices for this use. While the Two-Path digital filter seems to hold the most promise in this area, a further investigation of its operation (at radio frequencies) would be necessary before any conclusions may be drawn. Some suggestions for future work of this kind are included in the following section.

Suggestions for Future Study

Of primary interest for further study would be the operation of the Two-Path Digital Filter at frequencies within the standard broadcast band. While the same transient problem will exist for this device as for the four-path Digital Filter, it is conceivable that the keying and carrier balancing techniques used in the Two-Path device would minimize the problem.

Should operation at these frequencies prove feasible, the direct application of the Two-Path filter as the selectivity determining element in a communications receiver could be investigated. With the use of commercially available low-pass filters*, the Two-Path digital filter

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*Freed Transformer Co., Inc., 1795 Weirfield St., Brooklyn 27, N.Y.; ESC Electronics Corp., 534 Bergen Blvd., Palisades Park, N.J., and others.

could be made to have a shape factor rivaling that of the mechanical filters in common use today with the added feature of continuously adjustable center frequency. Simple RC low-pass sections might also be used with this device to construct the broadcast receiver envisioned by Texas Instruments (Chapter I).

An investigation might also be made into the possibility of using any of these devices as a tracking filter. That is, a filter which may be made to "lock on" to the input signal and hold it in the center of its passband even though the input signal may be drifting about its nominal frequency with time. Such a device might make possible the application of filters of a few cycles bandwidth to signals in the 100 megacycle region.

A SELECTED BIBLIOGRAPHY

(1) Langford-Smith, F., ed. Radiotron Designers Handbook. Section 9, Harrison, N. J.: Radio Corporation of America, 1953.

(2) Brown, J. N., and C. E. Weaver. "Crystal Lattice Filters for Transmitting and Receiving," Part 1, QST, Vol. 35, June, 1951, p. 48, and Part 2, August, 1951, p. 52.

(3) Roberts, Walter Van B. "Magnetoscriction Devices and Mechanical Filters for Radio Frequencies," Part 1, QST, Vol. 37, June, 1953, p. 24, Part 2, July, 1953, p. 28, and Part 3, August, 1953, p. 32.

(4) Recent Equipment, "Heathkit Mohican Transistorized Communications Receiver," QST, January, 1951, p. 11.

(5) Goodman, Byron. "What's Wrong With Our Present Receivers?" QST, Vol. 35, January, 1951, p. 11.

(6) Barber, N. F. "Narrow Band-Pass Filter Using Modulation," Wireless Engineer, Vol. 24, 1947, pp. 132-134.

(7) Madella, G. B. "Single Phase and Polyphase Filtering Devices Using Modulation," Wireless Engineer, Vol. 24, 1947, pp. 310-311.

(8) MacDiarmid, I. F., and D. G. Tucker. "Polyphase Modulation of Certain Problems in Telecommunications," Journal of the IEE, Vol. 97 - Part 3, 1950, pp. 349-358.

(9) Tucker, D. G. "Highly Selective Transmission Measuring Equipment for Communications Circuits," Journal of the IEE, Vol. 94 - Part 3, 1947, pp. 211-216.

(10) Paris, H. "Utilization of Quadrature Functions as a Unique Approach to Electronic Filter Design," IRE Convention Record, Part 9, 1960, pp. 204-216.

(11) Franks, L. E., and I. W. Sandburg. "An Alternative Approach to the Realization of Network Transfer Functions - The N-Path Filter," Bell System Technical Journal, Vol. 39, 1960, pp. 1321-1350.

(12) Smith, B. D. "Analysis of Commutated Networks," IRE Transactions on Aeronautical Engineering, ANE-10, 1953, pp. 21-26.

(13) Fischl, R. "Analysis of a Commutated Network," IEEE Transactions on Aerospace and Navigational Electronics, ANE-10, 1963, pp. 114-123.

(14) Brown, J. S., C. R. Cahn, and W. R. LePage. "Analysis of a Comb Filter Using Synchronously Commutated Capacitors," Transactions of the AIEE, Vol. 72, 1953, pp. 63-68.

(15) General Electric Company. Final Report on Digital Filter Research, New York, 1962.

(16) Acampora, A., B. Rabinovici, and C. A. Renton. "Low-Pass to Bandpass Transformation by Switching Techniques," RCA Engineer, Vol. 8, December, 1962-January, 1963, p. 41.

(17) Norgaard, Donald E. "The Phase Shift Method of Single Sideband Generation," Proceedings of the IRE, Vol. 44, 1956, pp. 1718-1735.

(18) Weaver, Donald K. "A Third Method of Generation and Detection of Single Sideband," Proceedings of the IRE, Vol. 44, 1956, pp. 1703-1705.

(19) Clarke, Kenneth R., and Maurice V. Joyce. Transistor Circuit Analysis, Reading, Massachusetts: Addison Wesley, 1962.

(20) General Electric Company. GE Transistor Manual, Sixth Edition. Syracuse, New York: General Electric Company, 1962.

(21) Millman, Jacob, and Herbert Taub. Pulse and Digital Circuits. New York: McGraw-Hill, 1956.

(22) Schwartz, Mischa. Information Transmission, Modulation and Noise. Chapters 3 and 4. New York: McGraw-Hill, 1959.

(23) Terman, Frederick E. Electronic and Radio Engineering, Fourth Edition, Chapters 15 and 16, New York: McGraw-Hill, 1953.

APPENDIX A

CALCULATION OF SIDEBAND SUPPRESSION DETERMINING CIRCUIT TOLERANCES

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CALCULATION OF SIDEBAND SUPPRESSION DETERMINING CIRCUIT TOLERANCES

A.1 Low-Pass Filter Amplitude Tolerances

As is shown in Chapter III, suppression of the unwanted sideband in the output signal of the Passband Generator is directly related to the matching of the I and Q Channels. Since the gain functions of the mixers and balanced modulators are essentially constants in this application (and hence, easily matched with gain controls), the channel matching is essentially dependent upon the matching of the low-pass filters used. If good sideband suppression is to be obtained throughout the entire passband of the generator, then the two low-pass filters must be matched to within some specified tolerance everywhere in their passbands.

The tolerance required for a specified amount of sideband suppression may be calculated as follows. Assume that the two low-pass filters have characteristics as shown in Figure A-1. At any given frequency, the gain of the I Channel may be designated by G_2 and the gain of the Q Channel by G_1. Since the signal amplitudes in each channel are directly proportional to G_1 and G_2, the amplitude of the desired output signal is given by $K(G_1 + G_2)$ and the amplitude of the undesired sideband is given as $K(G_1 - G_2)$ where K is some constant. The attenuation (A) of the undesired sideband with respect to the desired output signal is then given by

$$A_{db} = 20 \log \left| \frac{K(G_1 + G_2)}{K(G_1 - G_2)} \right|$$

$$= 20 \log \left| \frac{G_1 + G_2}{G_1 - G_2} \right| . \qquad \text{(A-1)}$$

To simplify calculations, this may be rewritten

$$A_{db} = 20 \log \left(\frac{1 + \frac{G_2}{G_1}}{1 - \frac{G_2}{G_1}} \right) =$$

$$= 20 \log \left(\frac{1 + k}{1 - k} \right) , \qquad 0 \leq k \leq 1 \quad \text{(A-2)}$$

where $k = G_2/G_1$ and where G_2 is always taken as the smaller of the two gains.

For example, if it is known that the two low-pass filters are everywhere matched to within 10%, then $G_2/G_1 = 0.9$ and

$$A_{db} = 20 \log \left(\frac{1 + 0.9}{1 - 0.9} \right) = 20 \log \frac{1.9}{0.1} = 20 \log (19) =$$

$$= 25.5 \text{ db.}$$

In other words, assuming that the filter is otherwise operating perfectly, the best suppression of the unwanted sideband that may be expected is about 25 decibels.

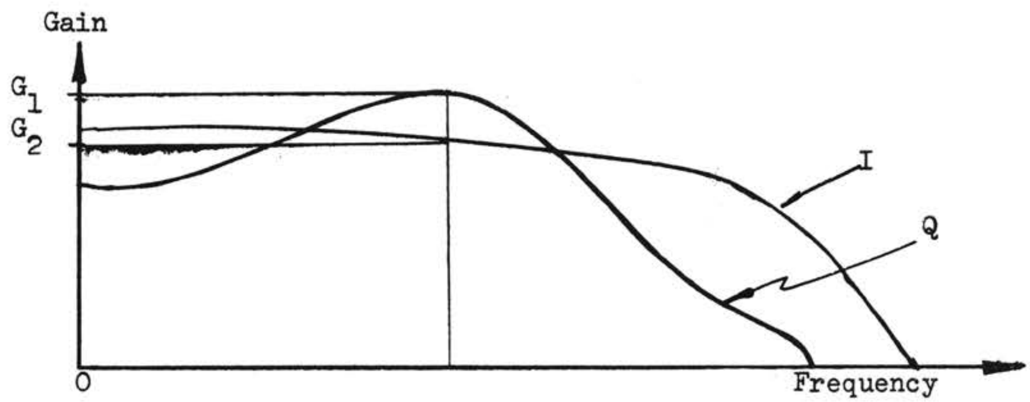

Figure A-1. Amplitude Characteristics of Two Low-Pass Filters

A curve may be plotted showing sideband suppression as a function of filter matching and is shown in Figure A-2. Notice that 60 decibels attenuation required channel matching to within 0.1%.

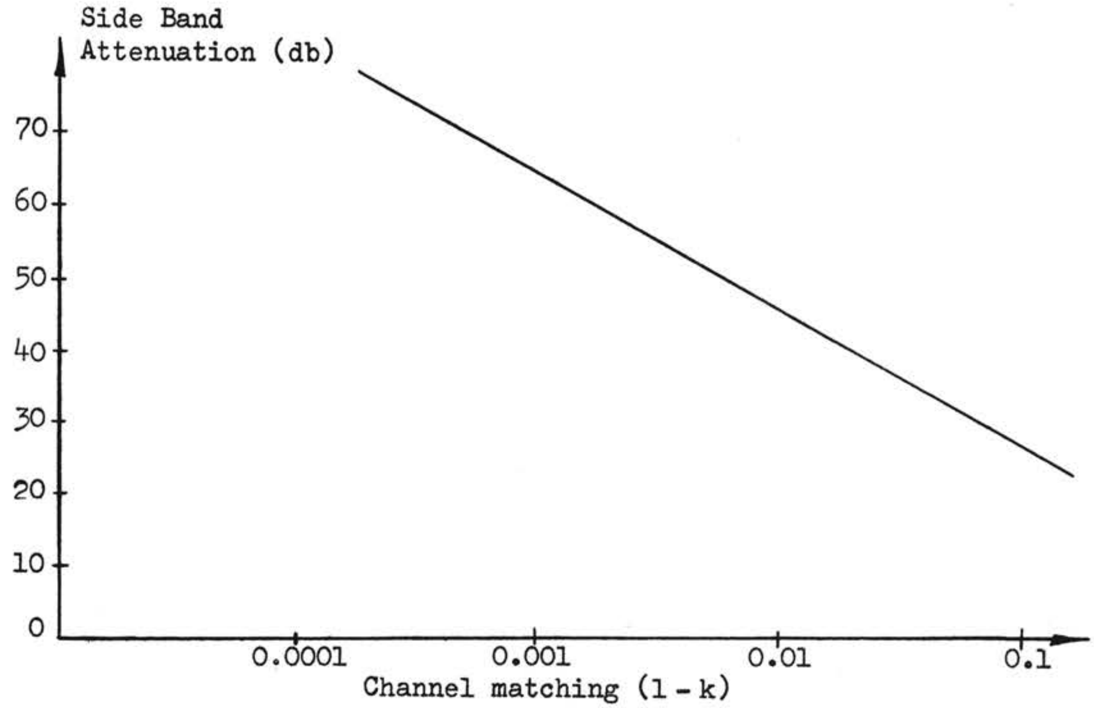

Figure A-2. Sideband Attenuation as a Function of Channel Matching

A.2 Ninety Degree Phase Tolerance

Calculating the allowable ninety degree phase tolerance for a given amount of sideband suppression requires returning to the analysis of Chapter III and modifying the expressions to include a phase angle increment φ.

Including this increment, the expressions for v_4 and v_4' (refer to Figure 3-1) may be written

$$v_4 = M \operatorname{Cos} \Delta\omega t \operatorname{Cos} \omega_o t$$

$$v_4' = M \operatorname{Cos} (\Delta\omega t - [90° + \varphi]) \operatorname{Cos}(\omega_o t + 90° + \varphi)$$

where M includes the mixer gain constants and the low-pass filter characteristics. It is assumed that the two channels have identical amplitude characteristics.

$$v_4 = \frac{M}{2}[\mathrm{Cos}\,(\omega_o t + \Delta\omega t) + \mathrm{Cos}\,(\omega_o t - \Delta\omega t)] \qquad \text{(A-3)}$$

and

$$v_4' = \frac{M}{2}[\mathrm{Cos}(\Delta\omega t - 90° - \varphi + \omega_o t + 90° + \varphi)$$

$$+ \mathrm{Cos}(\Delta\omega t - 90° - \varphi - \omega_o t - 90° - \varphi)]$$

$$v_4' = \frac{M}{2}[\mathrm{Cos}(\omega_o t + \Delta\omega t) + \mathrm{Cos}(\omega_o t - \Delta\omega t + 2\varphi)]. \qquad \text{(A-4)}$$

Notice that the phase increment in no way affects the $(\omega_o + \Delta\omega)t$ term, but appears doubled in the $(\omega_o - \Delta\omega)t$ term.

Now, since $v_5 = v_4 + v_4'$, it is found after simplifying that

$$v_5 = \frac{M}{2}[2\,\mathrm{Cos}\,(\omega_o t + \Delta\omega t) + \mathrm{Cos}\,(\omega_o t - \Delta\omega t)$$

$$- \mathrm{Cos}\,(\omega_o t - \Delta\omega t + 2\varphi)]. \qquad \text{(A-5)}$$

If φ were zero, notice that v_5 would be the output signal obtained in Chapter III.

By using the trigonometric identity

$$\mathrm{Cos}\,(A + B) = \mathrm{Cos}\,A\,\mathrm{Cos}\,B - \mathrm{Sin}\,A\,\mathrm{Sin}\,B\,. \qquad \text{(A-6)}$$

Equation A-5 may be rewritten

$$v_5 = \frac{M}{2}[2\,\mathrm{Cos}(\omega_o t + \Delta\omega t) + \mathrm{Cos}(\omega_o t - \Delta\omega t) - \mathrm{Cos}(\omega_o t - \Delta\omega t)\mathrm{Cos}\,2\varphi$$

$$+ \mathrm{Sin}\,(\omega_o t - \Delta\omega t)\mathrm{Sin}\,2\varphi] \qquad \text{(A-7)}$$

or

$$v_5 = \frac{M}{2}[2\text{Cos}(\omega_o t + \Delta\omega t) - (\text{Cos}[\omega_o t - \Delta\omega t][1 - \text{Cos }2\varphi] -$$

$$\text{Sin}[\omega_o t - \Delta\omega t]\,\text{Sin }2\varphi)] \quad . \qquad \text{(A-8)}$$

The last two terms in Equation A-8 are two sine waves ninety degrees out of phase; one with amplitude $(1 - \text{Cos } 2\varphi)$ and the other with amplitude $\text{Sin } 2\varphi$. These may be combined to form a single sine wave of amplitude

$$V = \sqrt{(1 - \text{Cos } 2\varphi)^2 + (\text{Sin } 2\varphi)^2} \ .$$

If only relative amplitudes are considered, it is then possible to obtain an expression relating φ to sideband suppression.

$$A_{db} = 20 \log \left| \frac{2}{\sqrt{(1 - \text{Cos } 2\varphi)^2 + (\text{Sin } 2\varphi)^2}} \right|$$

where the amplitude of the desired sideband is two units. This may be reduced to

$$A_{db} = 20 \log \left| \frac{\sqrt{2}}{\sqrt{1 - \text{Cos } 2\varphi}} \right| \quad . \qquad \text{(A-9)}$$

Using Equation A-9, the phase tolerances required for a given sideband suppression may now be computed. For example, if $\varphi = 1$ degree,

$$A_{db} = 20 \log \left| \frac{\sqrt{2}}{\sqrt{1 - \text{Cos } 2^\circ}} \right| = 20 \log 58.4 = 35.3 \text{ db.}$$

Similar calculations may be made for other values of φ and a plot made. Such a plot is shown in Figure A-4.

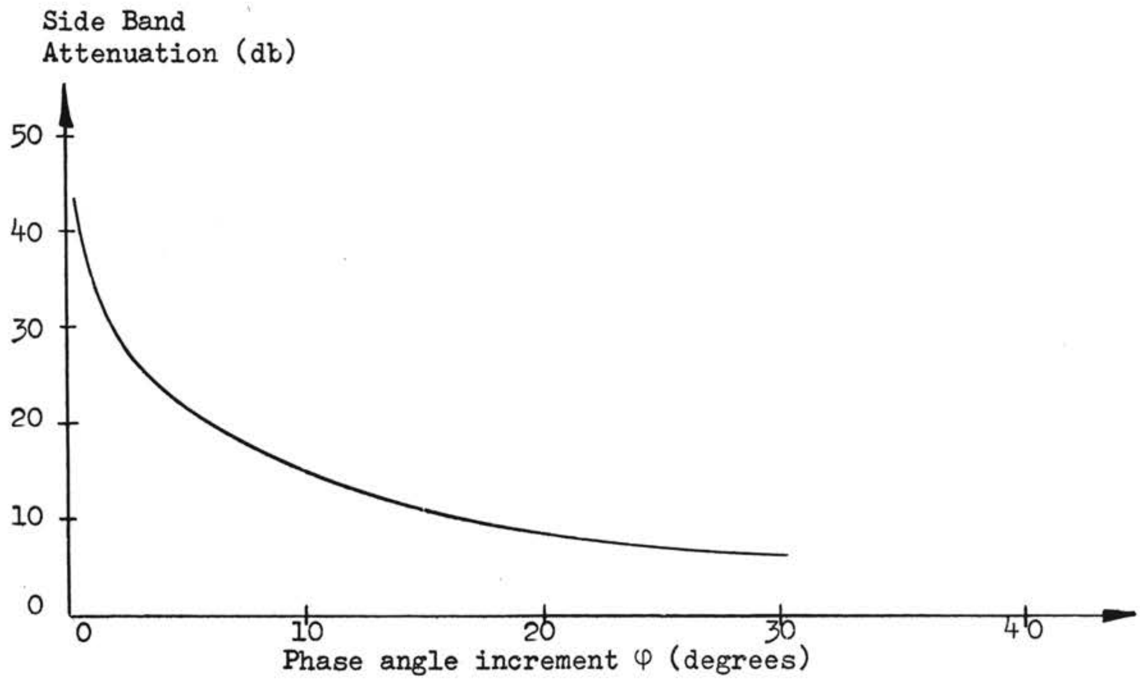

Figure A-3. Sideband Attenuation as a Function of Phase Increment φ

APPENDIX B

RESPONSE OF PASSBAND GENERATOR TO AN AMPLITUDE MODULATED SIGNAL

APPENDIX B

RESPONSE OF PASSBAND GENERATOR TO AN AMPLITUDE MODULATED SIGNAL

Consider the system of Figure 3-1 with an amplitude modulated signal given by

$$v_1 = V_1(1 + \text{Cos}\ \omega_m t)\ \text{Cos}(\omega_o + \Delta\omega t) \qquad \text{(B-1)}$$

applied to its input terminal. As in the analysis in Chapter III, $\Delta\omega$ is the difference between the center frequency of the filter and the input signal (or in this case, the carrier) frequency. The frequency of the modulating signal is given by ω_m. Since, for the purposes of analysis, it is desired that all sidebands pass through the generator, the assumption that $\Delta\omega + \omega_m < \omega_c$ is also made. These frequency relationships are demonstrated in Figure B-1.

Using Equation 3-3, the input signal may be written as

$$v_1 = V_1\ \text{Cos}\ (\omega_o + \Delta\omega)t + \frac{V_1}{2}\text{Cos}\ (\omega_o + \Delta\omega + \omega_m)t$$
$$+ \frac{V_1}{2}\text{Cos}\ (\omega_o + \Delta\omega - \omega_m)t\ . \qquad \text{(B-2)}$$

As before, the local oscillator signals supplied to the first mixers are given by $V_o Cos(\omega_o t)$ and $V_o Cos(\omega_o t + 90°)$. The two mixer outputs are then

$$v_2 = [K_A V_o Cos\ \omega_o t][v_1]$$

$$v_2' = [K_A' V_o Cos(\omega_o t + 90°)][v_1]\ .$$

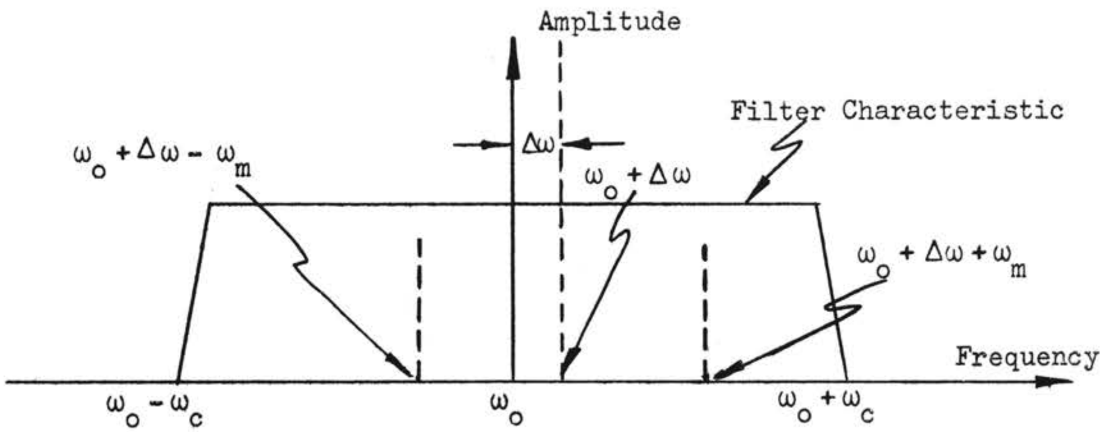

Figure B-1. Relationships of Various Signal Frequencies - Amplitude Modulated Input Signal

Substituting Equation B-2 for v_1 in the above and simplifying yields

$$v_2 = \frac{K_A V_o V_1}{2} Cos(2\omega_o + \Delta\omega)t + \frac{K_A V_o V_1}{2} Cos\,\Delta\omega t$$

$$+ \frac{K_A V_o V_1}{4} Cos(\Delta\omega + \omega_m)t + \frac{K_A V_o V_1}{4} Cos(2\omega_o + \Delta\omega + \omega_m)t$$

$$+ \frac{K_A V_o V_1}{4} Cos(2\omega_o + \Delta\omega - \omega_m)t + \frac{K_A V_o V_1}{4} Cos(\Delta\omega - \omega_m)t.$$

(B-3)

and

$$v_2' = \frac{K_A'V_oV_1}{2} \text{Cos } (-\Delta\omega t + 90°) + \frac{K_A'V_oV_1}{2} \text{Cos } (2\omega_o t + \Delta\omega t + 90°)$$

$$+ \frac{K_A'V_oV_1}{4} \text{Cos } (-\Delta\omega t - \omega_m t + 90°)$$

$$+ \frac{K_A'V_oV_1}{4} \text{Cos } (2\omega_o t + 90° + \Delta\omega t + \omega_m t)$$

$$+ \frac{K_A'V_oV_1}{4} \text{Cos } (-\Delta\omega t + \omega_m t + 90°)$$

$$+ \frac{K_A'V_oV_1}{4} \text{Cos } (2\omega_o t + 90° + \Delta\omega t - \omega_m t) \ . \tag{B-4}$$

After passing through the low-pass filters, this reduces to

$$v_3 = \frac{K_A F(\Delta\omega) V_o V_1}{2} [\text{Cos } \Delta\omega t + \tfrac{1}{2} \text{Cos } (\Delta\omega + \omega_m)t$$

$$+ \tfrac{1}{2} \text{Cos } (\Delta\omega - \omega_m)t] \tag{B-5}$$

and

$$v_3' = \frac{K_A' F'(\Delta\omega) V_o V_1}{2} [\text{Cos } (\Delta\omega t - 90°) + \tfrac{1}{2} \text{Cos}(\Delta\omega t + \omega_m t - 90°)$$

$$+ \tfrac{1}{2} \text{Cos } (\Delta\omega t - \omega_m t - 90°) \ . \tag{B-6}$$

These two voltages are now fed to the balanced modulators where they are remixed with the appropriate local oscillator signals yielding

$$v_4 = [v_3][V_o \mathrm{Cos}\, \omega_o t] K_B$$

$$= \frac{K_A F(\Delta\omega) K_B V_o^2 V_1}{4} [\mathrm{Cos}\,(\omega_o - \Delta\omega)t + \mathrm{Cos}(\omega_o + \Delta\omega)t$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o + \Delta\omega + \omega_m)t$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o - \Delta\omega - \omega_m)t$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o + \Delta\omega - \omega_m)t$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o - \Delta\omega + \omega_m)t] \qquad \text{(B-7)}$$

and

$$v_4' = [v_3][V_o \mathrm{Cos}\,(\omega_o t + 90°)] K_B'$$

$$v_4' = \frac{K_A' F'(\Delta\omega) K_B' V^2_o V_1}{4} [\mathrm{Cos}(\omega_o t - \Delta\omega t + 180°) + \mathrm{Cos}\,(\omega_o t + \Delta\omega t)$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o t - \Delta\omega t - \omega_m t + 180°)$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o t + \Delta\omega t + \omega_m t)$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o t - \Delta\omega t + \omega_m t + 180°)$$

$$+ \frac{1}{2}\mathrm{Cos}\,(\omega_o t + \Delta\omega_m t - \omega_m t)] \; . \qquad \text{(B-8)}$$

As in Chapter III, assume

$$M = \frac{F(\Delta\omega) K_A K_B}{4}$$

and

$$M' = \frac{F'(\Delta\omega) K_A' K_B'}{4} \; .$$

Then, since

$$v_5 = v_4 + v_4'$$

v_5 may be written as

$$v_5 = \frac{M}{2} \text{Cos } (\omega_o + \Delta\omega)t + \frac{M}{4} \text{Cos } (\omega_o + \Delta\omega + \omega_m)t$$

$$+ \frac{M}{4} \text{Cos } (\omega_o + \Delta\omega - \omega_m)t \qquad \text{(B-9)}$$

assuming that the two channels are identical ($M = M'$). This is exactly the input signal modified by the passband characteristic of the filter. For clarity, this may be written

$$v_5 = \frac{F(\Delta\omega)K_A K_B}{8} [\text{Cos } (\omega_o + \Delta\omega)t + \frac{1}{2} \text{Cos } (\omega_o + \Delta\omega + \omega_m)t$$

$$+ \frac{1}{2} \text{Cos } (\omega_o + \Delta\omega - \omega_m)t] \ . \qquad \text{(B-10)}$$

APPENDIX C

PASSBAND GENERATOR SET-UP PROCEDURE

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PASSBAND GENERATOR SET-UP PROCEDURE

The following is a description of a set-up technique for the Passband Generator which the author has found to be highly effective. A 40 kilocycle center frequency is assumed here but any other frequency in the 25 to 45 kilocycle range may be substituted if desired.

A 40 kilocycle signal at 100 millivolts rms is applied to the signal input of the Passband Generator and an eight volt (peak-to-peak) signal at 40 kilocycles is applied to the local oscillator input. Using an oscilloscope, the Q channel phase shifter is first set to ninety degrees. With the local oscillator signal applied to the horizontal input of the oscilloscope and the output of the phase shifter emitter follower (Q18) fed to the vertical input, the Phase Shift control (R_1) is set so that a circle appears on the oscilloscope face in the typical Lissajous figure.

The oscilloscope probe (vertical input) is then moved to the Generator output terminal (collector of Q_{23}) and the signal frequency adjusted to about 45 kilocycles. The oscilloscope sweep is then returned to its standard time base and the following adjustments made. The Q channel output is removed from the last adder (Q_{23}) by operating switch S_2 and the I Carrier Amplitude (R_2) and I Carrier Phase (R_5) controls adjusted for minimum local oscillator (carrier) signal. It should be possible to adjust the carrier level to less than one millivolt. The Q channel is then reconnected to the last adder by releasing S_2 and the I channel removed with switch S_1. The above process is repeated using the Q Carrier Amplitude (R_3) and Q Carrier Phase (R_6) controls. When the I Carrier Amplitude is reduced as far as possible it is reconnected to the output adder. Due to an interaction between the Carrier Amplitude and Carrier Phase controls, it will be necessary to alternately adjust these two potentiometers until the desired carrier minimum is obtained.

The signal frequency is now readjusted so that it is near the center of the passband. The Sideband Cancellation (R_4) and Ninety Degree Phase Shift (R_1) controls are then adjusted for minimum "noise" on the output signal. This noise is a beat between the local oscillator and the input signal and appears as modulation on the output signal. While the beat signal appears most readily as modulation with the oscilloscope sweep set to some low frequency, say 20 cycles per second, these adjustments are most easily made with the sweep frequency chosen so as to display two or three cycles of the output waveform. It is also worthwhile to peak the output tuned circuit at this time to insure it is centered on the passband of the generator and it is not contributing to the over-all selectivity.

With the noise minimized in this fashion, the signal frequency is again moved off the center frequency of the filter by about five kilocycles and the carrier noise canceled using all four of the carrier cancelling controls. The signal frequency is then returned to near the center of the passband and the Sideband Cancel and Ninety Degree controls repeaked. In some instances, after initial carrier balance is made as described above, final balance is most easily achieved by observing the output waveform with an input signal approximately 20 cycles per second off the center frequency applied to the input. Alternate adjustment of the two carrier level controls and the Sideband Cancel (R_4) and Ninety Degree Phase Shift (R_1) will yield essentially complete cancellation of the beat component of the output signal. This adjustment may not hold exactly elsewhere in the passband for reasons explained elsewhere.

In the way of a summary, it should be said that the purpose of these adjustments is to simultaneously cancel both the unwanted sideband and carrier from the output signal. Any procedure which achieves this result may, of course, be substituted for the above.

APPENDIX D

SET-UP PROCEDURE FOR TWO-PATH DIGITAL FILTER

APPENDIX D

SET-UP PROCEDURE FOR TWO-PATH DIGITAL FILTER

This procedure bears a close similarity to the procedure discussed in Appendix C and the reader will be referred there for some of the steps.

Assuming that the ninety degree relationship between the two carrier signals has been determined as described in Appendix C and that the square wave voltages at the collectors of Q_9 and Q_{20} have been connected (using ac coupling) to the switching transistor bases of the Two-Path filter set up proceeds as follows. The input signal is tuned out of the passband and the two carrier balance controls adjusted for minimum signal out of the filter. The input signal is then returned to a point approximately 20 cycles per second off the center frequency of the filter and the ninety degree phase control adjusted for minimum "noise" on the signal. The carrier balance controls may be touched up at this time also completing the adjustment procedure.

VITA

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Candidate for the Degree of

Master of Science

Thesis: AN INVESTIGATION OF THREE SAMPLED-DATA BANDPASS FILTERS

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